Department of Civil Engineering

Regulation 2021
III Year – V Semester
CE3502/ Structural Analysis I

CE3502 **STRUCTURAL ANALYSIS I** **L T P C**

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COURSE OBJECTIVE:

1. To introduce the students to the basic theory and concepts of analysis of trusses
2. To introduce the students to the basic theory and concepts of slope deflection method
3. To learn the principles of moment distribution method
4. To learn the principles of matrix flexiblity method
5. To introduce the students to the basic theory and concepts of matrix stiffness method

UNIT I **ANALYSIS OF TRUSSES** **9**

Determinate and indeterminate trusses - analysis of determinate trusses - method of joints - method of sections - Deflections of pin-jointed plane frames - lack of fit - change in temperature method of tension coefficient - Application to space trusses.

UNIT II **SLOPE DEFLECTION METHOD** **9**

Slope deflection equations – Equilibrium conditions - Analysis of continuous beams and rigid frames – Rigid frames with inclined members - Support settlements - symmetric frames with symmetric and skew-symmetric loadings.

UNIT III **MOMENT DISTRIBUTION METHOD** **9**

Stiffness - distribution and carry over factors –- Analysis of continuous Beams- Plane rigid frames with and without sway – Support settlement - symmetric frames with symmetric and skew-symmetric loadings.

UNIT IV **FLEXIBLITY METHOD** **9**

Primary structures - Compatibility conditions – Formation flexibility matrices - Analysis of indeterminate pin- jointed plane frames, continuous beams and rigid jointed plane frames by direct flexibility approach.

UNIT V **STIFFNESS METHOD** **9**

Restrained structure –Formation of stiffness matrices - equilibrium condition - Analysis of Continuous Beams, Pin-jointed plane frames and rigid frames by direct stiffness method.

TOTAL: 45 PERIODS

COURSE OUTCOMES:

Students will be able to

CO1 Analyze the pin-jointed plane and space frames.

CO2 Analyse the continuous beams and rigid frames by slope defection method.

CO3 Understand the concept of moment distribution and analysis of continuous beams and rigid frames with and without sway.

CO4 Analyse the indeterminate pin jointed plane frames continuous beams and rigid frames using matrix flexibility method.

CO5 Understand the concept of matrix stiffness method and analysis of continuous beams, pin jointed trusses and rigid plane frames.

TEXTBOOKS:

1. Bhavikatti, S.S,Structural Analysis,Vol.1,& 2, Vikas Publishing House Pvt.Ltd.New Delhi-4, 2014.

2. Punmia.B.C, Ashok Kumar Jain & Arun Kumar Jain, Theory of structures, Laxmi Publications, New Delhi, 2004.

REFERENCES:

1. William Weaver, Jr and James M.Gere, Matrix analysis of framed structures, CBS Publishers & Distributors, Second Edition, Delhi, 2004

2. Reddy .C.S, "Basic Structural Analysis", Tata McGraw Hill Publishing Company, 2005.

3. Negi L.S. and Jangid R.S., Structural Analysis, Tata McGraw Hill Publishing. Co. Ltd. 2004

4. Bhavikatti, S.S, Matrix Method of Structural Analysis, I. K. International Publishing House Pvt.Ltd.,New Delhi-4, 2014.

UNIT – I

ANALYSIS OF TRUSSES

1. What are the assumptions made in analysis of a pin-joined plane truss?

The assumptions made in analysis of a pin-joined plane truss are
a) The frame is a perfect frame.
b) The frame carries a load at the joints.
c) All the members are pin-jointed

2. What are the methods available for the analysis of a frame?

The following are the methods available for the analysis of a frame:
a) Methods of joints
b) Methods of sections
c) Graphical method

3. What are the assumptions made in finding out the forces in a frame?

The assumptions made in finding out the forces in a frame are:
a) The frame is perfect
b) The frame carries load at the joints
c) All the members are pin-jointed

4. What is tension coefficient?

The force per unit length of a member is known as tension coefficient. T = F/ L where T is tension coefficient F is force and L is length of the member.

5. What are Deficient and Redundant frames?

If the number of members are more than (2j – 3), then the frame is known as redundant frame.

6. Define internally and externally indeterminate structures.

In a pin jointed frames redundancy caused by too many members is called internally indeterminate structures or internal redundancy.

In a pin jointed frames redundancy caused by too many supports is called externally indeterminate structures or external redundancy.

7. Differentiate perfect and imperfect frame.

A structural frame that is stable under loads imposed upon it from any direction is known as perfect frame. A structural frame is unstable if one of its members were removed or one of its fixed ends became hinged is known as imperfect frame.

8. Define degree of freedom.

In a structure the number of independent joint displacement that the structures can undergo are known as degree of freedom. It is also known as kinematic indeterminacy.

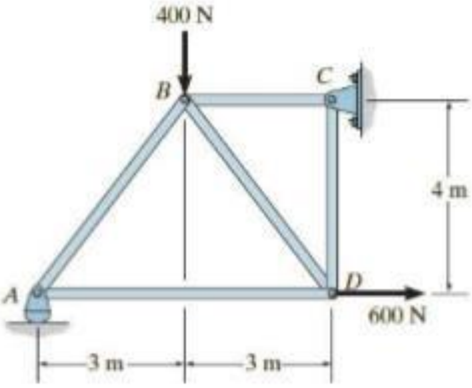
9. Define static indeterminacy of structures.

If the conditions of statics i.e. $\sum H = 0$, $\sum V = 0$ and $\sum M = 0$ alone are not sufficient to find either external reactions or internal forces in a structure. The structure is called static indeterminacy of structures.

10. Define static determinate structures.

If the conditions of statics i.e. $\sum H = 0$, $\sum V = 0$ and $\sum M = 0$ alone are sufficient to find either external reactions or internal forces in a structure. The structure is called static determinacy of structures.

11. Determine the force in each member of the truss shown in Figure. Indicate whether the members are in tension or compression.



Taking moment of all forces about C

$V_A \times 6 - 400 \times 3 - 600 \times 4 = 0$

$V_A = 600$ N

Now $\sum F_X = 0$

=> $-H_C + 600 = 0$

=> $H_C = 600$ N

Now $\sum F_Y = 0$

=> $600 - 400 - V_C = 0$

=> $V_C = 200$ N

At Joint A:

$\sum F_Y = 0$

$600 + F_{AB} \times 4/5 = 0$

=>$F_{AB} = -750$ N (C)

$\sum F_X = 0$

$F_{AD} - 750 \times 3/5 = 0$

=>$F_{AD} = 450$ N (T)

At Joint D:

$\sum F_X = 0$

$-F_{DB} \times 3/5 - 450 + 600 = 0$

$\Rightarrow F_{DB} = 250$ N (T)

$\sum F_Y = 0$

$F_{DC} + 250 \times 4/5 = 0$

$\Rightarrow F_{DC} = -200$ N (C)

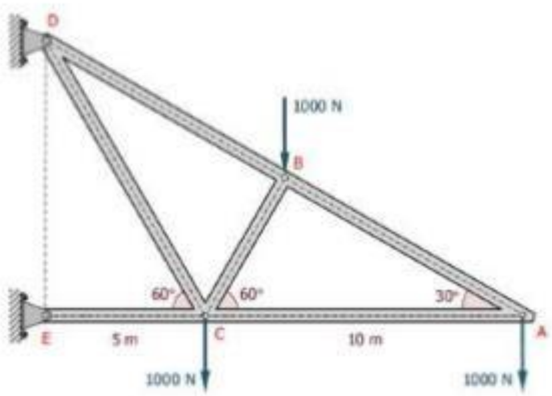
At Joint C:

$\sum F_X = 0$

$-F_{CB} - 600 = 0$

$\Rightarrow F_{CB} = -600$ N (C)

12. The cantilever truss in Figure is hinged at D and E. Find the force in each member.



Taking moment of all forces about D

$H_E \times 8.66 - 1000 \times 5 - 1000 \times 7.5 - 1000 \times 15 = 0$

$H_E = 3175$ N

Now $\sum F_X = 0$

$\Rightarrow H_D + 3175 = 0$

$\Rightarrow H_D = -3175$ N

At Joint A:

$\sum F_Y = 0$

$-1000 + F_{AB} \times \sin 30° = 0$

$\Rightarrow F_{AB} = 2000$ N (T)

$\sum F_X = 0$

$F_{AC} - 2000 \times \cos 30° = 0$

$=> F_{AC} = 1732$ N (C)

At Joint B:

$\sum F_Y = 0$

$- 1000 - F_{AB} \times \sin30° + F_{BD} \times \sin30° - F_{BC} \times \sin60° = 0$

$\sum F_X = 0$

$F_{AB} \times \cos30° - F_{BD} \times \cos30° - F_{BC} \times \cos60° = 0$

$=> F_{BD} = 2500$ N (T)

$=> F_{BC} = 866$ N (C)

At Joint C:

$\sum F_Y = 0$

$- 1000 + F_{CD} \times \sin60° + F_{CB} \times \sin60° = 0$

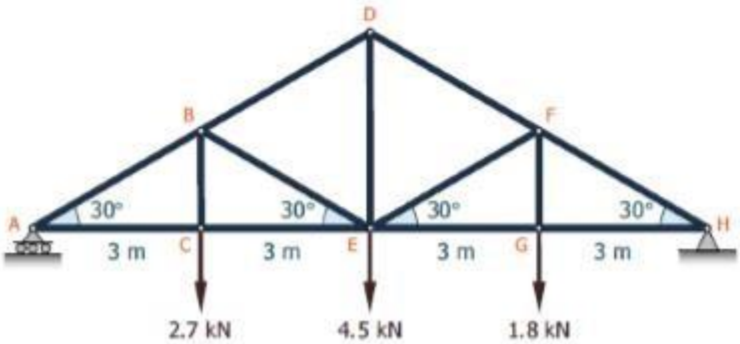
$\sum F_X = 0$

$F_{AC} - F_{CE} - F_{CD} \times \cos60° + F_{CB} \times \cos60° = 0$

$=> F_{CD} = 2020$ N (T)

$=> F_{CE} = 3175$ N (C)

13. Determine the force in members AB, BD, BE, and DE of the Howe roof truss shown in Figure.



Taking moment of all forces about A

$V_H \times 12 – 2.7 \times 3 – 4.5 \times 6 – 1.8 \times 9 = 0$

$V_H = 4.275$ N

Now $\sum F_Y = 0$

$=> 9 – V_C = 0$

$=> V_A = 4.725$ N

At Joint A:

$\sum F_Y = 0$

$4.725 + F_{AB} \times \sin 30° = 0$

$\Rightarrow F_{AB} = 9.45kN$ (C)

At Joint B:

$\sum F_Y = 0$

$2.7 + F_{AB} \times \sin 30° + F_{BD} \times \sin 30° - F_{BE} \times \sin 30° = 0$

$\sum F_X = 0$

$F_{AB} \times \cos 30° - F_{BD} \times \cos 30° - F_{BE} \times \cos 30° = 0$

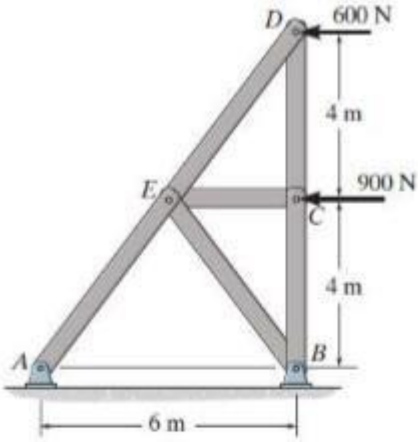
$\Rightarrow F_{BD} = 6.75kN$ (C)

$\Rightarrow F_{BE} = 2.7kN$ (C)

At Joint D$\sum F_Y = 0$

$F_{DE} = 6.75$ kN(T)

14. Determine the force in each member of the truss, and state if the members are in tension or compression.



At Joint D:

$\sum F_X = 0$

$- 600 + F_{DE} \times 3/5 = 0$

$\Rightarrow F_{DE} = 1000$ N (C)

$\sum F_Y = 0$

$F_{DC} - 1000 \times 4/5 = 0$

$\Rightarrow F_{DC} = 800$ N (T)

At Joint C:

$\sum F_X = 0$

=>F_{CE} = 900 N (C)

$\sum F_Y = 0$

=>F_{CB} = 800 N (T)

At Joint E:

$\sum F_X = 0$ and $\sum F_Y = 0$

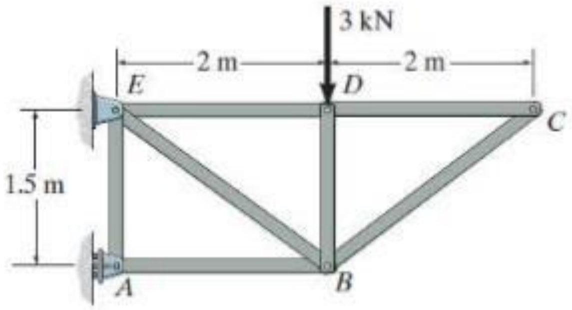
$F_{EA} \times 3/5 + F_{ED} \times 3/5 + F_{EB} \times 3/5 - 900 = 0$

$F_{EA} \times 4/5 + F_{ED} \times 4/5 + F_{EB} \times 4/5 - 900 = 0$

=>F_{EB} = 750 N (T)

=>F_{EA} = 1750 N (C)

15. Determine the force in each member of the truss, and state if the members are in tension or compression.



At Joint C:

$\sum F_X = 0$

=>F_{CD} = 0 N

$\sum F_Y = 0$

$F_{CB} \times 1.5/2.5 - 0 = 0$

=>F_{CB} = 0 N

At Joint D:

$\sum F_X = 0$

=>F_{DE} = 0 N

$\sum F_Y = 0$

=>F_{DB} = 3kN (T)

At Joint B:

$\sum F_X = 0$ and $\sum F_Y = 0$

$F_{BA} \times 1.5/2.5 + F_{BE} \times 1.5/2.5 = 0$

$F_{BA} \times 2/2.5 + F_{BE} \times 2/2.5 - 3 = 0$

$\Rightarrow F_{BE} = 4.5$ N (C)

$\Rightarrow F_{BA} = 3$ N (T)

At Joint A: $\sum F_Y = 0$

$\Rightarrow F_{AE} = 0$ N

UNIT – II

SLOPE DEFLECTION METHOD

1. What are the assumptions made in slope-deflection method?

a) Between each pair of the supports the beam section is constant.
b) The joint in structure may rotate or deflect as a whole, but the angles between the members meeting at that joint remain the same.

2. What is the limitation of slope-deflection equations applied in structural analysis?

a) It is not easy to account for varying member sections.
b) It becomes very cumbersome when the unknown displacements are large in number.

3. Explain the use of slope deflection method.

a) It can be used to analyze statically determinate and indeterminate beams and frames.
b) In this method it is assumed that all deformations are due to bending only.
c) In other words deformations due to axial forces are neglected.
d) The slope-deflection equations are not that lengthy in comparison.

4. How many slope deflection equations are available for a two span continuous beam?

There will be 4 number of slope deflection equations, two for each span.

5. What is the moment at a hinged end of a simple beam?

Moment at the hinged ends of a simple beam is zero.

6. What are the quantities in terms of which the unknown moments are expressed in slope deflection method?

In slope-deflection method, unknown moments are expressed in terms of (i) Slopes and (ii) Deflections.

7. Mention any three reasons due to which sway may occur in portal frames.

Sway in portal frames may occur due to
i) Unsymmetry in geometry of the frame
ii) Unsymmetry in loading
iii) Settlement of one end of a frame.

8. A rigid frame is having totally 10 joints including support joints. Out of slope deflection and moment distribution methods, which method would you prefer for analysis? Why?

Moment distribution method is preferable. If we use slope-deflection method, there would be 10 (or more) unknown displacements and an equal number of equilibrium equations. In addition, there would be 2 unknown support moments per span and the same number of slope-deflection equations. Solving them is difficult.

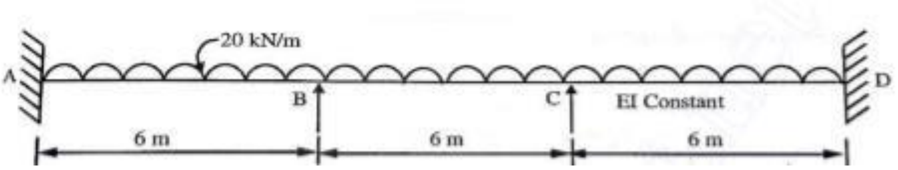
9. What is a couple?

The turning effect produced by two equal and opposite force separated by a distance constitute a couple.

10. What are the quantities in terms of which the unknown moments are expressed in slope deflection method?

In slope-deflection method, unknown moments are expressed in terms of (i) Slopes and (ii) Deflections.

11. Analyse the continuous beam shown in figure. Calculate the support moments using slope deflection method.



Fixed End Moments:

$MF_{AB} = MF_{BC} = MF_{CD} = -Wl^2/12 = -20x6^2/12 = -60$ kNm

$MF_{BA} = MF_{CB} = MF_{DC} = Wl^2/12 = 20x6^2/12 = 60$ kNm

Slope Deflection Equations:

The structure is symmetrical. So is the loading. Hence the following conditions prevail.

$\theta_A = \theta_D = \delta = 0$

$\theta_B = \theta_C$ Hence there is only one unknown displacement, namely θ_B.

For span AB, the general slope deflection equation is

$M_{AB} = MF_{AB} + 2EI/6(2\theta_A + \theta_B + 3\delta/l)$

$M_{AB} = -60 + 2EI/6(\theta_B)$ since $\theta_A = 0$ and $\delta = 0$

$M_{BA} = 60 + 2EI/6(2\theta_B)$

For span BC,

$M_{BC} = MF_{BC} + 2EI/6(2\theta_B + \theta_C + 3\delta/l)$

$M_{BC} = -60 + 2EI/6(3\theta_B)$

Joint Equilibrium Equations:

$M_{BA} + M_{BC} = 0$

$60 + 2EI\theta_B/3 - 60 + EI\theta_B = 0$

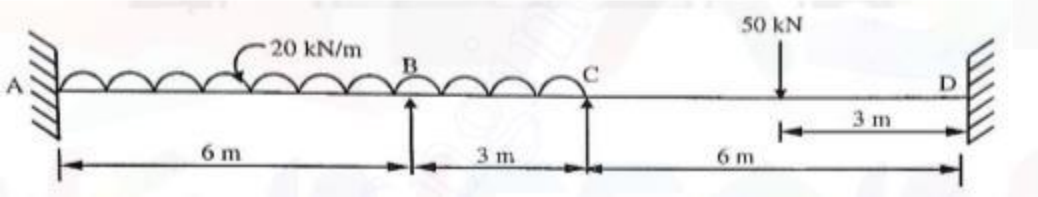
Hence, $\theta_B = 0$

Final Moments:

$M_{AB} = M_{BC} = M_{CD} = -60$ kNm

$M_{BA} = M_{CB} = M_{DC} = 60$ kNm

12. Analyse the continuous beam shown in figure. Calculate support moments using slope deflection method. Support B sinks by 10mm. Take E = 2 x 10^5 N/mm^2,I = 16 x 10^7mm^4.



Fixed End Moments:

$MF_{AB} = -Wl^2/12 = -20x6^2/12 = -60$ kNm

$MF_{BA} = Wl^2/12 = 20x6^2/12 = 60$ kNm

$MF_{BC} = -Wl^2/12 = -20x3^2/12 = -15$kNm

$MF_{CB} = Wl^2/12 = 20x3^2/12 = 15$kNm

$MF_{CD} = -Wl/8 = -50x6/8 = -37.5$kNm

$MF_{DC} = Wl/8 = 50x6/8 = 37.5$kNm

Slope Deflection Equations:

$M_{AB} = MF_{AB} + 2EI/6(2\theta_A + \theta_B + 3\delta/l)$

$M_{AB} = -60 + 2EI/6(0 + \theta_B - 1/200)$

$M_{BA} = MF_{BA} + 2EI/6(2\theta_B + \theta_A + 3\delta/l)$

$M_{BA} = 60 + 2EI/6(2\theta_B - 1/200)$

$M_{BC} = MF_{BC} + 2EI/3(2\theta_B + \theta_C + 3\delta/l)$

$M_{BC} = -15 + 2EI/3(2\theta_B + \theta_C + 1/100)$

$M_{CB} = MF_{CB} + 2EI/3(2\theta_C + \theta_B + 3\delta/l)$

$M_{CB} = 15 + 2EI/3(2\theta_C + \theta_B + 1/100)$

$M_{CD} = MF_{CD} + 2EI/3(2\theta_C + \theta_D + 3\delta/l)$

$M_{CD} = -37.5 + 2EI/3(2\theta_C + 0)$

$M_{DC} = MF_{DC} + 2EI/3(2\theta_D + \theta_C + 3\delta/l)$

$M_{DC} = 37.5 + 2EI/3(0 + \theta_C)$

Joint Equilibrium Equations:

$M_{BA} + M_{BC} = 0$

$EI/3(6\theta_B + 2\theta_C + 3/200) = -135$

$M_{CB} + M_{CD} = 0$

$EI(\theta_B + 3\theta_C + 1/100) = 33.75$

Solving Joint Equilibrium Equations we get

$\theta_C = - 1/464; \theta_B = - 1/402$

Final Moments:

$M_{AB} = - 139.843$ kNm

$M_{BA} = - 46.354$ kNm

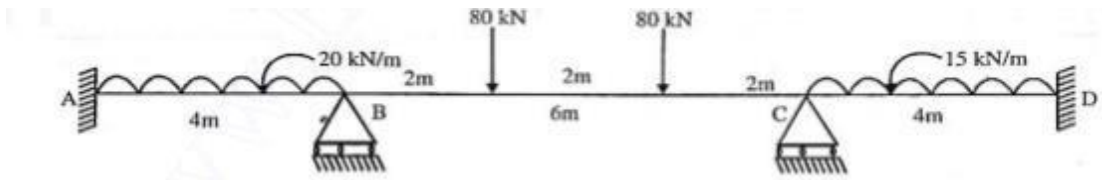
$M_{BC} = 46.3$ kNm

$M_{CB} = 83.35$ kNm

$M_{CD} = - 83.477$ kNm

$M_{DC} = 14.51$ kNm

13. Analyse the continuous beam shown in figure. Calculate the support moments using slope deflection method. Take $2I_{AB} = I_{BC} = 2I_{CD} = 2I$.



Fixed End Moments:

$MF_{AB} = -Wl^2/12 = - 20x4^2/12 = - 26.67$kNm

$MF_{BA} = Wl^2/12 = 20x4^2/12 = 26.67$kNm

$MF_{BC} = -Wa(a + c)/6 = - 80(4 + 2)/6 = - 106.67$ kNm

$MF_{CB} = Wa(a + c)/6 = 80(4 + 2)/6 = 106.67$ kNm

$MF_{CD} = -Wl^2/12 = - 15x4^2/12 = - 20$ kNm

$MF_{DC} = Wl^2/12 = 15x4^2/12 = 20$ kNm

Slope Deflection Equations:

$M_{AB} = MF_{AB} + 2EI/4(2\theta_A + \theta_B + 3\delta/l)$

$M_{AB} = - 26.67 + 2EI/4(0 + \theta_B)$

$M_{BA} = MF_{BA} + 2EI/4(2\theta_B + \theta_A + 3\delta/l)$

$M_{BA} = 26.67 + 2EI/4(2\theta_B)$

$M_{BC} = MF_{BC} + 2EI/6(2\theta_B + \theta_C + 3\delta/l)$

$M_{BC} = -106.67 + 2EI/6(2\theta_B + \theta_C)$

$M_{CB} = MF_{CB} + 2EI/6(2\theta_C + \theta_B + 3\delta/l)$

M_{CB} = 106.67 + 2EI/6(2θ_C + θ_B)

M_{CD} = MF_{CD} + 2EI/4(2θ_C + θ_D + 3δ/l)

M_{CD} = -20 + 2EI/4(2θ_C + 0)

M_{DC} = MF_{DC} + 2EI/4(2θ_D + θ_C + 3δ/l)

M_{DC} = 20 + 2EI/4(0 + θ_C)

Joint Equilibrium Equations:

M_{BA} + M_{BC} = 0

EI(2.333θ_B + 0.666θ_C+) = 80

M_{CB} + M_{CD} = 0

EI(0.666θ_B + 2.333θ_C) = 86.67

Solving Joint Equilibrium Equations we get

θ_C = - 51.11/EI; θ_B = 48.88/EI

Final Moments:

M_{AB} = - 2.23 kNm

M_{BA} = 75.554 kNm

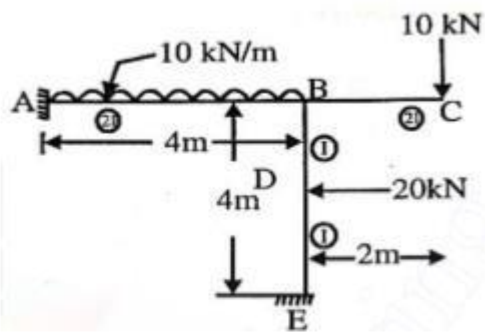
M_{BC} = - 75.554 kNm

M_{CB} = 71.06 kNm

M_{CD} = - 71.06 kNm

M_{DC} = 5.56kNm

14. Analyse the rigid frame shown in figure. Calculate the support moments using slope deflection method.



Fixed End Moments:

MF_{AB} = -Wl^2/12 = - 20x4^2/12 = - 13.33kNm

MF_{BA} = Wl^2/12 = 20x4^2/12 = 13.33kNm

MF_{BC} = - 10 × 2 = - 20kNm

MF_{BE} = -Wl/8 = - 15x4^2/12 = - 10 kNm

MF_{EB} = Wl/8 = 15x4^2/12 = 10 kNm

Slope Deflection Equations:

$M_{AB} = MF_{AB} + 2EI/4(2\theta_A + \theta_B + 3\delta/l)$

$M_{AB} =- 13.33+ EI\theta_B$

$M_{BA}= MF_{BA} + 2EI/4(2\theta_B + \theta_A + 3\delta/l)$

$M_{BA}=13.33+ EI\theta_B$

$M_{BE}= MF_{CD} + 2EI/4(2\theta_C + \theta_D + 3\delta/l)$

$M_{BE}= -10 + EI\theta_B$

$M_{EB}= MF_{DC} + 2EI/4(2\theta_D + \theta_C + 3\delta/l)$

$M_{EB}=10 +0.5EI\theta_B$

Joint Equilibrium Equations:

$M_{BA} + M_{BC} + M_{BE} = 0$

$13.33 + EI\theta_B - 10 + EI\theta_B - 20 = 0$

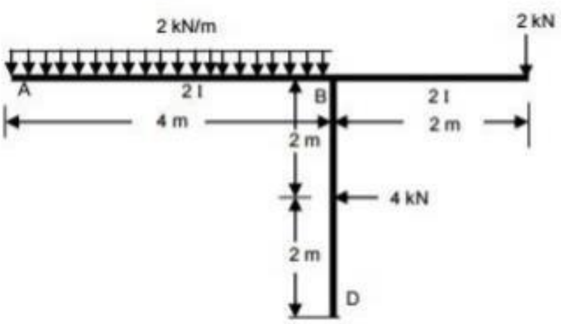
$\theta_B = 5.55/EI$

Final Moments:

M_{AB} = - 7.73 kNm; M_{BA} = 24.554 kNm; M_{BC} = - 20kNm

M_{CD} = - 4.06 kNm; M_{DC} = 12.56 kNm

15. Analyse the rigid frame shown in figure. Calculate the support moments using slope deflection method.



Fixed End Moments:

MF_{AB} = -Wl^2/12 = - 2x4^2/12 = - 2.67kNm

MF_{BA} = Wl^2/12 = 2x4^2/12 = 2.67kNm

$MF_{BC} = -2 \times 2 = -4kNm$

$MF_{BD} = -Wl/8 = -4\times4/8 = -2kNm$

$MF_{DB} = Wl/8 = 4\times4/8 = 2kNm$

Slope Deflection Equations:

$M_{AB} = MF_{AB} + 2EI/4(2\theta_A + \theta_B + 3\delta/l)$

$M_{AB} = -2.667 + EI\theta_B$

$M_{BA} = MF_{BA} + 2EI/4(2\theta_B + \theta_A + 3\delta/l)$

$M_{BA} = 2.667 + 2EI\theta_B$

$MF_{BD} = MF_{CD} + 2EI/4(2\theta_C + \theta_D + 3\delta/l)$

$MF_{BD} = -2 + EI\theta_B$

$MF_{DB} = MF_{DC} + 2EI/4(2\theta_D + \theta_C + 3\delta/l)$

$MF_{DB} = 2 + 0.5EI\theta_B$

Joint Equilibrium Equations:

$M_{BA} + M_{BC} + MF_{BD} = 0$

$2.667 + 2EI\theta_B - 4 + 2 + 0.5EI\theta_B = 0$

$\theta_B = 1.11/EI$

Final Moments:

$M_{AB} = -1.56kNm$; $M_{BA} = 4.554$ kNm; $M_{BC} = -4kNm$

$M_{CD} = -0.86$ kNm; $M_{DC} = 2.56$ kNm

UNIT – III

MOMENT DISTRIBUTION METHOD

1. Differentiate between distribution factors and carry over factor.

Distribution factor: When several members meet at a joint and a moment is applied at the joint to produce rotation without translation of the members, the moment is distributed among all the members meeting at that joint proportionate to their stiffness.

Distribution factor = Relative stiffness / Sum of relative stiffness at the joint

Carry over factor: A moment applied at the hinged end B "carries over" to the fixed end A, a moment equal to half the amount of applied moment and of the same rotational sense.

2. Define point of contra flexure with an example.

In a bending moment diagram, where the sign changes from positive to negative or negative to positive that place is called point of contra flexure.

3. Define flexural rigidity.

The product of young's modulus (E) and moment of inertia (I) is called Flexural Rigidity (EI) of Beams. The unit is N / mm^2.

4. What is sway correction? Explain.

Sway correction is defined as the removal of lateral movement in the beams or frames by correction factor is multiplied by corresponding sway moment.

5. What is distribution factor?

When several members meet at a joint and a moment is applied at the joint to produce rotation without translation of the members, the moment is distributed among all the members meeting at that joint proportionate to their stiffness.

6. What is stiffness of a prismatic member?

The stiffness of a prismatic member is 4EI / L.

7. Explain the relative stiffness factor.

Relative stiffness is the ratio of stiffness to two or more members at a joint.

8. In a member AB, if a moment of -10 kNm is applied at A what is the moment carried over to B?

Carry over moment = Half of the applied moment

Carry over moment to B = -10/5 = -5 KNm.

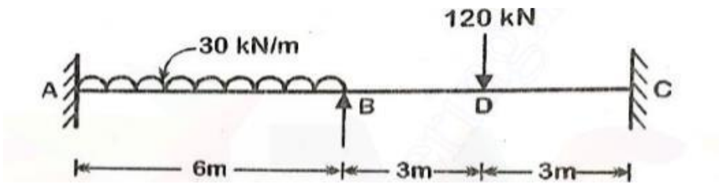
9. What is the sum of distribution factors at a joint?

Sum of distribution factors at a joint = 1.

10. Define the term sway.

Sway is the lateral movement of joints in a portal frame due to the unsymmetry in dimensions, loads, moments of inertia, end conditions, etc.

11. For the continuous beam shown in figure, calculate the support moments by moment distribution method.



Fixed End Moments:

MF_{AB} = $-Wl^2/12$ = - $30x6^2/12$ = - 90 kNm

MF_{BA}= $Wl^2/12$ = $30x6^2/12$ = 90 kNm

MF_{BC} = -Wl/8 = - 120×6/8 = -90 kNm

MF_{CB} = Wl/8 = 120×6/8 = 90kNm

Distribution Factor Table:

Joint	Member	k	∑k	Distribution factor (k/∑k)
B	BA	4EI/6	4EI/3	0.5
	BC	4EI/6		0.5

Moment Distribution Table:

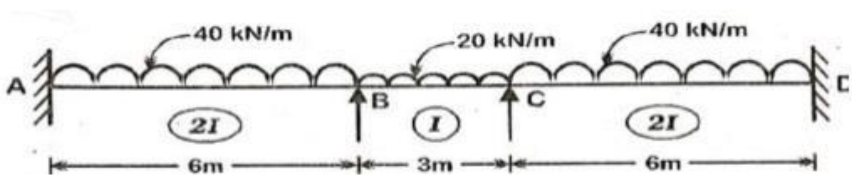
Joint	A	B		C
Member	AB	BA	BC	CB
D F	-	0.5	0.5	-
FEM	-90	90	-90	90
Distribution	-	0	0	-
Final Moments	-90	90	-90	90

Final Moments:

M_{AB} = - 90 kNm; M_{BA} = 90 kNm;

M_{BC} = - 90 kNm; M_{CD} = 90 kNm

12. For the continuous beam shown in figure find the support moment by moment distribution method. Carry out two cycles of distribution.



Fixed End Moments:

MF_{AB} = $-Wl^2/12$ = - $40x6^2/12$ = - 120 kNm

MF_{BA}= $Wl^2/12$ = $40x6^2/12$ = 120 kNm

MF_{BC} = $-Wl^2/12$ = - $20x3^2/12$ = - 15kNm

$MF_{CB} = Wl^2/12 = 20x3^2/12 = 15kNm$

$MF_{CD} = -Wl^2/12 = -40x6^2/12 = -120$ kNm

$MF_{DC} = Wl^2/12 = 40x6^2/12 = 120$ kNm

Distribution Factor Table:

Joint	Member	k	∑k	Distribution factor (k/∑k)
B	BA	4EI/3	7EI/3	0.57
	BC	3EI/3		0.43
C	CB	3EI/3	7EI/3	0.43
	CD	4EI/3		0.57

Moment Distribution Table:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D F	-	0.57	0.43	.43	.57	-
FEM	-120	120	-15	15	-120	120
Distribution		-59.9	-45.1	45.1	59.9	
Carry over	-29.95		22.6	-22.6		29.95
Distribution		-12.9	-9.7	9.7	12.9	
Carry over	-6.45					6.45
Final Moments	-156.4	47.2	-47.2	47.2	-47.2	156.4

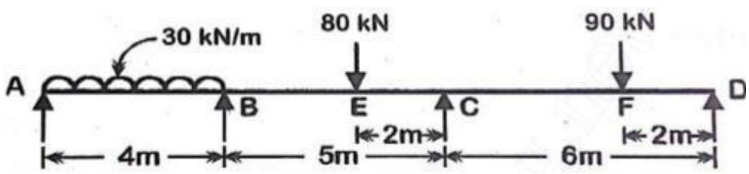
Final Moments:

$M_{AB} = -156.4$ kNm; $M_{BA} = 47.2$ kNm

$M_{BC} = -47.2$ kNm; $M_{CB} = 47.2$ kNm

$M_{CD} = -47.2$ kNm; $M_{DC} = 156.4$ kNm

13. A continuous beam ABCD simply supported at A,B,C and D is loaded as shown in figure. Calculate the support moments by moment distribution method. EI is constant.



Fixed End Moments:

$MF_{AB} = -Wl^2/12 = -30\times4^2/12 = -40$ kNm

$MF_{BA} = Wl^2/12 = 30\times4^2/12 = 40$ kNm

$MF_{BC} = -Wab^2/l^2 = -80\times3\times2^2/5^2 = -38.4kNm$

$MF_{CB} = -Wa^2b/l^2 = 80\times3^2\times2/5^2 = 57.6kNm$

$MF_{CD} = -Wab^2/l^2 = -90\times4\times2^2/5^2 = -40$ kNm

MF_{DC} = $-Wa^2b/l^2 = 90\times4^2\times2/5^2 = 80$ kNm

Distribution Factor Table:

Joint	Member	k	∑k	Distribution factor (k/∑k)
B	BA	3EI/4	27EI/20	0.56
	BC	3EI/5		0.44
C	CB	3EI/5	11EI/10	0.5
	CD	3EI/6		0.5

Moment Distribution Table:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D F	-	0.56	0.44	0.5	0.5	-
FEM	-40	40	-38.4	57.6	-40	80
Release	40					-80
Carry over		20			-40	
Initial Moment	0	60	-38.4	57.6	-80	0
Distribution		-12.09	9.5	11.2	11.2	
Carry over		0	5.6	-4.5	0	
Distribution		-3.13	-2.4	2.37	2.37	
Carry over			1.18	-1.23		
Distribution		-0.66	-0.52	0.61	0.61	
Carry over			0.30	-0.26		
Final	0	44	-44	66	-66	0

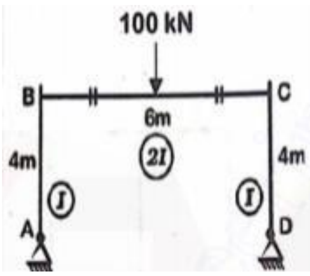
Final Moments:

M_{AB} = 0 kNm; M_{BA} = 44 kNm

M_{BC} = - 44 kNm; M_{CB} = 66 kNm

M_{CD} = - 66 kNm; M_{DC} = 0 kNm

14. For the rigid portal frame compute the support moments by moment distribution method.



Fixed End Moments:

MF_{AB} = 0

MF_{BA} = 0

MF_{BC} = -Wl/8= - 100×6/8 = - 75 kNm

MF_{CB} = -Wl/8= 100×6/8 = 75kNm

MF_{CD} = 0

MF_{DC} = 0

Distribution Factor Table:

Joint	Member	k	∑k	Distribution factor (k/∑k)
B	BA	3EI/4	7EI/4	0.43
	BC	6EI/6		0.57
C	CB	6EI/6	7EI/4	0.57
	CD	3EI/4		0.43

Moment Distribution Table:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D F	-	0.43	0.57	0.57	0.43	-
FEM			-75	75		
Distribution		32.25	42.75	-42.75	32.25	
Final Moment	0	32.5	-32.5	32.5	-32.5	0

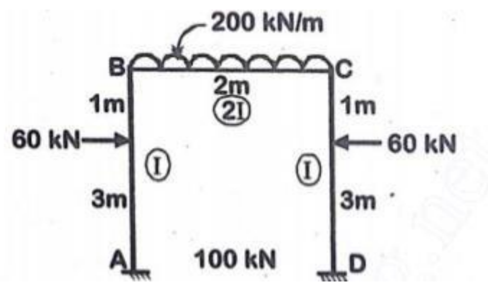
Final Moments:

M_{AB} = 0 kNm; M_{BA} = 32.5 kNm

M_{BC} = - 32.5 kNm; M_{CB} = 32.5 kNm

M_{CD} = - 32.5 kNm; M_{DC} = 0 kNm

15. Analyse the portal frames shown in figure by moment distribution method.



Fixed End Moments:

$MF_{AB} = -Wab^2/l^2 = -\ 60\times3\times1^2/4^2 = -\ 11.25$kNm

$MF_{BA} = Wa^2b/l^2 = 60\times3^2\times1/4^2 = 33.75$kNm

$MF_{BC} = -Wl^2/12 = -\ 200\times2^2/12 = -\ 66.67$kNm

$MF_{CB} = Wl^2/12 = 200\times2^2/12 = 66.67$kNm

$MF_{CD} = -Wab^2/l^2 = -\ 60\times1\times3^2/4^2 = -\ 33.75$kNm

$MF_{DC} = Wa^2b/l^2 = 60\times1^2\times3/4^2 = 11.25$kNm

Distribution Factor Table:

Joint	Member	k	∑k	Distribution factor (k/∑k)
B	BA	4EI/4	4EI	0.25
	BC	6EI/2		0.75
C	CB	6EI/2	4EI	0.75
	CD	4EI/4		0.25

Moment Distribution Table:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D F	-	0.25	0.75	0.75	0.25	-
FEM	-11.25	33.75	-66.67	66.67	-33.75	11.25
Distribution		8.23	24.69	-24.69	-8.23	
Carry over	4.11					-4.11
Final Moment	-7.14	41.98	-41.98	41.98	-41.98	7.14

Final Moments:

M_{AB} = -7.14 kNm; M_{BA} = 41.98 kNm

M_{BC} = - 41.98 kNm; M_{CB} = 41.98 kNm

M_{CD} = 41.98 kNm; M_{DC} = 7.14 kNm

UNIT – IV

FLEXIBLITY METHOD

1. Define kinematic redundancy.

When a structure is subjected to loads, each joint will undergo displacements in the form of translations and rotations. Kinematic redundancy of a structure means the number of unknown joint displacement in a structure.

2. Give the mathematical expression for the degree of static indeterminacy of rigid jointed plane frames.

Degree of static indeterminacy = (No. of closed loops x 3) – No. of releases

3. What are the conditions to be satisfied for determinate structures and how are indeterminate structures identified?

Determinate structures can be solving using conditions of equilibrium alone (H = 0; V = 0; M = 0). No other conditions are required.

Indeterminate structures cannot be solved using conditions of equilibrium because (H ≠ 0; V ≠ 0; M ≠ 0). Additional conditions are required for solving such structures.

4. Mention any two methods of determining the joint deflection of a perfect frame.

a) Unit load method
b) Virtual work method
c) Slope deflection method
d) Strain energy method

5. What are the requirements to be satisfied while analyzing a structure?

The three conditions to be satisfied are:

a) Equilibrium condition
b) Compatibility condition
c) Force displacement condition

6. What is meant by force method in structural analysis?

A method in which the forces are treated as unknowns is known as force method. The following are the force methods:

a) Flexibility matrix method
b) Consistent deformation method
c) Claypeyron’s 3 moment method
d) Column analogy method

7. Define flexibility coefficient.

It is defined as the displacement at coordinate i due to unit force at coordinate j in a structure. It makeup the elements of a flexibility matrix.

8. Why flexibility method is also called as compatibility method or force method?

Flexibility method begins with the superposition of forces and is hence known as force method. Flexibility method leads to equations of displacement compatibility and is hence known as compatibility method.

Introduction

The structure of a building (or other object) is the part which is responsible for maintaining the shape of the building under the influence of the forces, loads and other environmental factorstowhichitissubjected.Itisimportantthatthestructureasawhole(oranypartofit) does not fall down, break or deform to an unacceptable degree when subjected to such forces or loads. The study of structures involves the analysis of the forces and stresses occurring within a structure and the design of suitable components to cater for such forces and stresses.

As an analogy, consider the human body. Human body comprises a skeleton of 206 bones which constitutes the structure of human body. If any of those bones were to break, or if any of the joints between those bones were to disconnect or seize up, the injured body would'fail' structurally (and cause a great deal of pain).

Examplesofstructuralcomponents(or'members',asEngineersandArchitectscallthem) include:
• steelbeams,columns,rooftrussesandspaceframes;
• reinforcedconcretebeams,columns,slabs,retainingwallsandfoundations;
• timberjoists,columns,glulambeamsandrooftrusses;
• masonrywallsandcolumns.

Whatisanengineer?

The word 'engineer' comes from the French word ingénieur, which refers to someone who uses his/her ingenuity to solve problems. An engineer is a problem-solver.

A structural engineer solves the problem of ensuring that a building – or other structure – is adequate (in terms of strength, stability, cost, etc.) for its intended use.

Thestructuralengineerinthecontextofrelatedprofessions

someoftheprofessionalsinvolvedinthedesignofbuildingsincludethefollowing:

• thearchitect;
• thestructuralengineer;
• thequantitysurveyor.

Of course, this is not an exhaustive list. There are many other professionals involved in building design (for example, building surveyors and project managers) and many more trades and professions involved in the actual construction of buildings.

Thearchitectisresponsibleforthedesignofabuildingwithparticularregardtoits appearance and environmental qualities such as light levels and noise insulation.

The structural engineer is responsible for ensuring that the building can safely withstand all the forces to which it is likely to be subjected, and that it will not deflect or crack unduly in use.
The quantity surveyor is responsible for measuring and pricing the work to be undertaken – and for keeping track of costs as the work proceeds.

So,in short:
(1) Thearchitectmakessurethebuildinglooksgood.
(2) The(structural)engineerensuresitwillstandup.

(3) Thequantitysurveyorensuresitsconstructioniseconomical.

Structuralunderstanding

The basic function of a structure is to transmit loads from the position of application of the load to the point of support and thus to the foundations in the ground. Let us for the time being consider a load as being any force acting externally on a structure.)

Anystructuremustsatisfythefollowingcriteria:
(1) Aesthetics(itmustlooknice).
(2) Economy(itmustn'tcostmorethantheclientcanafford–andlessifpossible).
(3) Easeofmaintenance.
(4) Durability.Thismeansthatthematerialsusedmustberesistanttocorrosion,spalling (pieces falling off), chemical attack, rot or insect attack.
(5) Fireresistance.Whilefewmaterialscancompletelyresisttheeffectsoffire,itis important for a building to resist fire long enough for its occupants to be safely evacuated.

In order to ensure that a structure behaves in this way, one need to develop an understanding and awareness of how the structure works.

Safetyandserviceability

There are two main requirements of any structure: it must be safe and it must be serviceable. 'Safe' means that the structure should not collapse – either in whole or in part. 'Serviceable' means that thestructureshould not deform undulyunder the effects of deflection, cracking or vibration.

Safety

Astructuremustcarrytheexpectedloadswithoutcollapsingasawholeandwithoutany part of it collapsing. Safety in this respect depends on two factors:

(1) Theloadingthestructureisdesignedtocarryhasbeencorrectlyassessed.
(2) Thestrengthofthematerialsusedinthestructurehasnotdeteriorated.

From this it is evident that one needs to know how to determine the load on any part of a structure. Furthermore, One also needs to know that materials deteriorate in time if not properlymaintained: steel corrodes, concrete mayspall or suffer carbonation, timber will rot. The structural engineer must consider this when designing any particular building.
Serviceability

A structure must be designed in such a waythat it doesn't deflect or crack undulyin use. It is difficult or impossible to completely eliminate these things – the important thing is that the deflection and cracking are kept within certain limits. It must also be ensured that vibration does not have an adverse effect on the structure – this is particularly important in parts of buildings containing plant or machinery.

If, when one walks across the floor of a building, one feels the floor deflect or 'give' underneath one's feet, it may lead one to be concerned about the integrity of the structure. Excessivedeflectiondoesnotnecessarilymeanthatthefloorisabouttocollapse,butbecause it may lead to such concerns, deflection must be 'controlled'; in other words, it must be kept within certain limits. To take another example, if a lintel above a doorway deflects too much, it may cause warping of the door frame below it and, consequently, the door itself may not open or close properly. Cracking is ugly and may or may not be indicative of a structural problem. But it may, in itself, lead to problems. For example, if cracking occurs on the outside face of a reinforced concrete wall then rain may penetrate and cause corrosion of the steel reinforcement within the concrete.

Compositionofabuildingstructure

A building structure contains various elements, the adequacy of each of which is the responsibility of the structural engineer.

A roof protects people and equipment in a building from weather. Walls can have one ormore of several functions. The most obvious one is load bearing – in other words, supporting any walls, floors or roofs above it. But not all walls are load bearing. Other functions of awall include the following:

- partitioning, or dividing, rooms within a building – and thus defining their shape and extent;
- weatherproofing;
- thermalinsulation–keepingheatin(orout);
- noiseinsulation–keepingnoiseout(orin);
- fireresistance;
- securityandprivacy;
- resistinglateral(horizontal)loadssuchasthoseduetoretainedearth,windorwater.

A floor provides support for the occupants, furniture and equipment in a building. Floors on an upper level of a building are always suspended, which means that they span between supporting walls or beams. Ground floor slabs may sit directly on the ground beneath.

Staircasesprovideforverticalmovementbetweendifferentlevelsinabuilding.

Foundations represent the interface between the building's structure and the ground beneath it. A foundation transmits all the loads from a building into the ground in such a way that settlement (particularly uneven settlement) of the building is limited and failure of the underlying soil is avoided.

In a building it is frequently necessary to support floors or walls without any interruption or division of the space below. In this case, a horizontal element called a beam will be used. A beam transmits the loads it supports to columns or walls at the beam's ends.

A column is a vertical loadbearing element which usually supports beams and/or other columns above. Laymen often call them pillars or poles or posts. Individual elements of a structure, such as beams or columns, are often referred to as members.

Needtolearnaboutstructures

If one is studying architecture, one may be wondering why one needs to study structures at all. However, as an architect, it is important that one understands the principles of structural behaviour. On larger projects architects work in inter-disciplinary teams which usually include structural engineers. It is therefore important to understand about structural engineering. Remember – if one has difficulty in getting one's model to stand up, it is unlikely that the real thing will stand up either!

Basicaspectsofstructures

Structural engineers use the following words (amongst others, of course) in technical discussions:

• force
• reaction
• stress
• moment.

None of these words is new to any bode; they are all common English words that are used in everyday speech. However, in structural engineering each of these words has a particular meaning.

Force

A force is an influence on an object (for example, part of a building) that may cause movement. For example, the weight of people and furniture within a building causes a vertically downwards force on the floor, and wind blowing against a building causes a horizontal (or near horizontal) force on the external wall of the building.

Reaction

If one stands on a floor, the weight of your body will produce a downward force into the floor. The floor reacts to this by pushing upwards with a force of the same magnitude as the downward force due to your body weight.

This upward force is called a reaction, as its very presence is a response to the downward force of your body. Similarly, a wall or a column supporting a beam will produce an upward reaction as aresponseto thedownward forces the beam transmits to thewall (orcolumn) and a foundation will produce an upward reaction to the downward force in the column or wall that the foundation is supporting.

The same is true of horizontal forces and reactions. If one pushes horizontally against a wall, one's body is applying a horizontal force to the wall – which the wall will oppose with a horizontal reaction.

Stress

Stress is internal pressure. A heavy vehicle parked on a road is applying pressure to the road surface – the heavier the vehicle and the smaller the contact area between the vehicle's tyres and the road, the greater the pressure.

As a consequence of this pressure on the road surface, the parts of the road below the surface will experience a pressure which, because it is within an object (in this case, the road) is termed a stress. Because the effect of the vehicle's weight is likelyto be spread, or dispersed, as it is transmitted downwards within the road structure, the stress (internal pressure at a point) will decrease the further down you go within the road's construction. So, stress is internal pressure at a given point within, for example, a beam, slab or column. It is likelythat the intensity of the stress will vary from point to point within the object. Stress is a very important concept in structural engineering.

Moment

A moment is a turning effect. When one uses a spanner to tighten a nut, mechanically windup a clock or turn the steering wheel on one's car, one is applying a moment.

Howdostructuresorpartsofstructurebehave

Compression

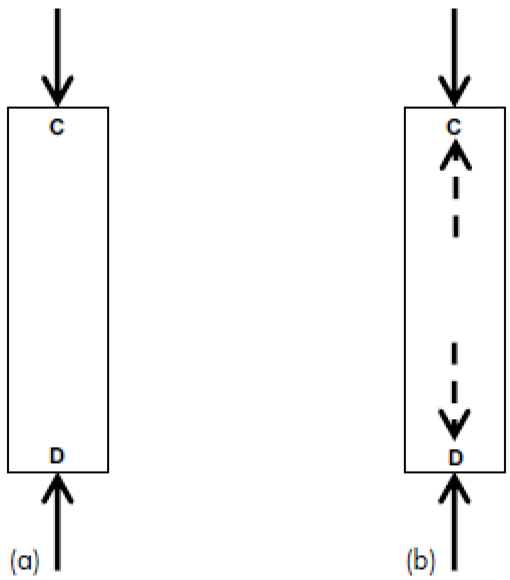


Figure1.1Acolumnincompression

Figure 1.1 (a) shows an elevation – that is, a side-on view – of a concrete column in a building. The column is supporting beams, floor slabs and other columns above and the load, or force, from all of these is acting downwards at the top of the column. This load is represented by the downward arrow at the top of the column. Intuitively, we know that the column is being squashed by this applied load – it is experiencing compression.

A downward force must be opposed by an equal upward force (or reaction) if the building is stationary– as it should be. This reaction is represented bythe upward arrow at the bottom of the column in Fig. 1.1 (a). Now, not only must the rules of equilibrium (total force up = total force down) apply for the column as a whole; these rules must apply at any and every point within a stationary structure.

Let's consider what happens at the top of the column – specifically, point C in Fig. 1.1 (b). The downward force shown in Fig. 1.1 (a) at point C must be opposed by an upward force – also at point C. Thus there will be an upward force within the column at this point, as represented by the upward broken arrow in Fig. 1.1 (b). Now let's consider what happens at theverybottomofthecolumn–pointDinFig.1.1(b).TheupwardforceshowninFig.1.1
(a) at point D must be opposed bya downward force at the same point. This is represented by the downward broken arrow in Fig. 1.1 (b).

Look at the direction of the broken arrows in Fig. 1.1 (b). These arrows represent the internal forces in the column. One will notice that they are pointing away from each other. This is always the case when a structural element is in compression: the arrows used to denote compression point away from each other.

Tension

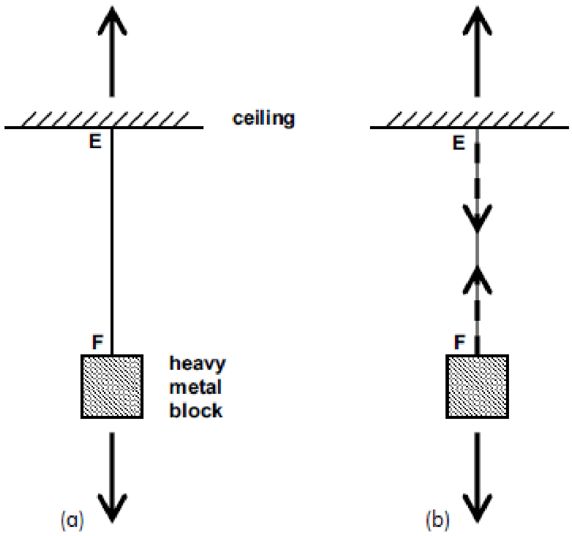


Figure1.2Apieceofstringintension

Figure 1.2 shows a heavy metal block suspended from the ceiling of a room by a piece of string. The metal block, under the effects of gravity, is pulling the string downwards, as represented by the downward arrow. The string is thus being stretched and is therefore in tension.

For equilibrium, this downward force must be opposed by an equal upward force at the point where the string is fixed to the ceiling. This opposing force is represented by an upwardarrow in Fig. 1.2 (a). Note that if the ceiling wasn't strong enough to carry the weight of the metal block, or the string was improperly tied to it, the weight would come crashing to the ground and there would be no upward force (or reaction) at this point. As with the column considered above, the rules of equilibrium (total force up = total force down) must apply at any and every point within this system if it is stationary.

Let's consider what happens at the top of the string. The upward force shown in Fig. 1.2 (a)at point E must be opposed by a downward force – also at this point. Thus there will be a downward force within the string at this point, as represented bythe downward broken arrow in Fig. 1.2 (b).

Now let's consider what happens at the very bottom of the string – at the point where the metal block is attached (point F). The downward force shown in Fig. 1.2 (a) at point F must be opposed by an upward force at this point. This upward force within the string at this point is represented by the upward broken arrow in Fig. 1.2 (b).

Look at the direction of the broken arrows in Fig. 1.2 (b). These arrows represent the internal forces in the string. One will notice that they are pointing towards each other. This is always the case when a structural element is in tension: the arrows used to denote tension point towards each other. (An easy way to remember this principle is the letter T, which stands for both Towards and Tension.)

The standard arrow notations for members in (a) tension and (b) compression are shown in Fig. 1.3.

Note:Tension andcompressionareboth examplesofaxialforces –theyact alongtheaxis(or centre line) of the structural member concerned.

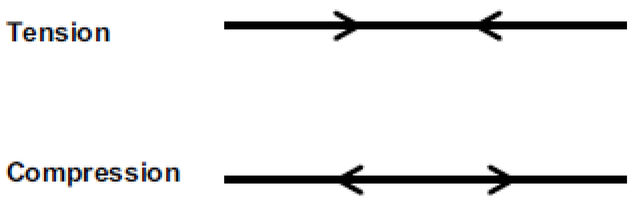


Figure1.3Arrownotationsfortensionandcompression

Bending

Considerasimplysupportedbeam(thatis,abeamthatsimplyrestsonsupportsatitstwo ends) subjected to a central point load. The beam will tend to bend, as shown in Fig. 1.4.

Theextenttowhichthebeambendswilldependonfourthings:
(1) The material from which the beam is made. One would expect a beam made of rubber to bend more than a concrete beam of the same dimensions under a given load.

(2) Thecross-sectionalcharacteristicsofthebeam.Alargediameterwoodentreetrunkis more difficult to bend than a thin twig spanning the same distance.

(3) Thespanofthebeam.Anyonewhohasevertriedtoputupbookshelvesathomewill know that the shelves will sag to an unacceptable degree if not supported at regular intervals. (The same applies to the hanger rail inside a wardrobe. The rail will sag noticeably under the weight of all those clothes if it is not supported centrally as well as at its ends.)

(4) The load to which the beam is subjected. The greater the load, the greater the bending. The bookshelves will sag to a greater extent under the weight of heavy encyclopedias than they would under the weight of a few light paperback books.

If One carries on increasing the loading, the beam will eventuallybreak. Clearly, the stronger thematerial,themore difficultitistobreak.Atimberrulerisquite easyto breakbybending;

a steel ruler of similar dimensions might bend quite readily but it’s unlikely that one wouldmanage to break it with your bare hands!

This is evidentlyonewayin which a beam can fail – through excessivebending. Beams must be designed so that they do not fail in this way.

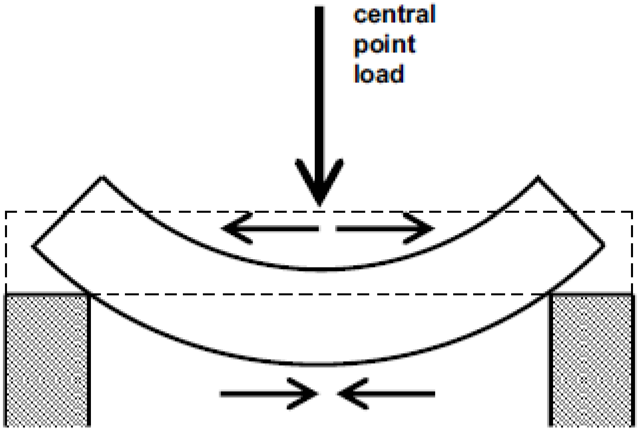
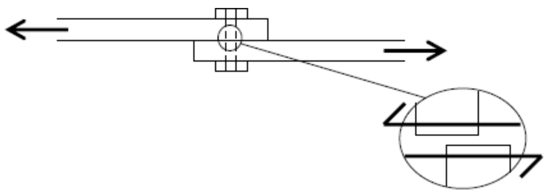
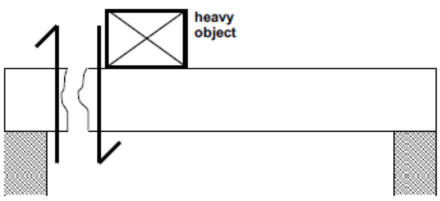


Figure1.4Bendinginabeam.

Shear



(a) Shear in a bolt connecting two plates



(b) Shear in a timber joist

Figure1.5Conceptofshear

Consider two steel plates that overlap each other slightly, with a bolt connecting the two plates through the overlapping part, as shown in Fig. 1.5 (a). Imagine now that a force is applied to the top plate, trying to pull it to the left. An equal force is applied to the bottom plate, trying to pull it to the right. Let’s now suppose that the leftward force is slowly increased, as is the rightward force. (Remember that the two forces must be equal if thewhole system is to remain stationary.) If the bolt is not as strong as the plates, eventually we will reach a point when the bolt will break. After the bolt has broken, the top part of it will move off to the left with the top plate and the bottom part will move off to the right with the bottom plate.

Let’sexamineindetailwhathappenstothefailuresurfaces(thatis,thebottomfaceofthetop part of the bolt and the top face of the bottom part of the bolt) immediately after failure. As you can see from the ‘exploded’ part of Fig. 1.5 (a), the two failure surfaces are sliding past each other. This is characteristic of a shear failure.

We’llnowturnourattentiontoatimberjoistsupportingthefirstfloorofabuilding,as showninFig.1.5(b).Let’simaginethattimberjoistsaresupportedonmasonry wallsand thatthejoiststhemselvessupportfloorboards,aswouldbethecaseinatypicaldomestic dwelling – such as, perhaps, the house you live in. Suppose that the joists are inappropriately undersized – in other words, they are not strong enough for the loads they are likely to haveto support.

Now let’s examine what would happen if a heavy object – for example, some large piece of machinery – was placed on the floor near its supports, as shown in Fig. 1.5 (b). If the heavy object is near the supporting walls, the joists may not bend unduly. However, if the object is heavyenough and the joists are weak enough, the joist maysimplybreak. This type of failure is analogous to the bolt failure discussed above. With reference to Fig. 1.5 (b), the right-hand part of the beam will move downwards (as it crashes to the ground), while the left-hand part of the beam will stay put – in other words, it moves upwards relative to the downward-moving right-hand part of a beam. So, once again, we get a failure where the two failure surfaces are sliding past each other: a shear failure. So a shear failure can be thought of as a cutting or slicing action. So, this is a second way in which a beam can fail – through shear. Beams must be designed so that they do not fail in this way. (Incidentally, the half-headed arrow notation shown in Fig. 1.5 is the standard symbol used to denote shear.)

StructuralElementsandtheirbehaviour Beams

Beams may be simply-supported, continuous or cantilevered, as illustrated in Fig. 1.6. They are subjected to bending and shear under load, and the deformations under loading are shown by broken lines.

A simply-supported beam rests on supports, usually located at each end of the beam. A continuous beam spans two or more spans in one unbroken unit; it may simply rest on its supports, but more usually it is gripped (or fixed) by columns above and below it. A cantilever beam is supported at one end only; to avoid collapse, the beam must be continuous over, or rigidly fixed at, this support.

Beamsmaybeoftimber,steelorreinforcedorprestressedconcrete.

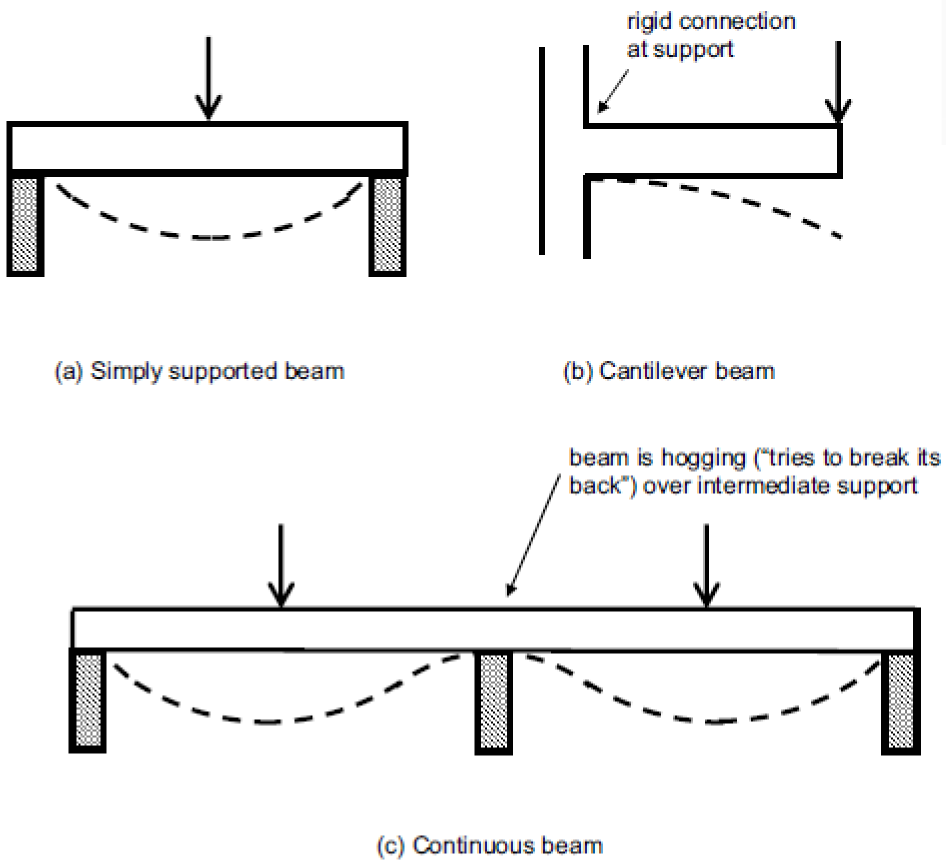


Figure1.6Beamtypes

Slabs

As with beams, slabs span horizontally between supports and may be simply supported, continuous or cantilevered. But unlike beams, which are usually narrow compared with their depth, slabs are usually wide and relatively shallow and are designed to form flooring – see Fig. 1.7.

Slabs may be one-way spanning, which means they are supported by walls on opposite sides of the slab, or two-way spanning, which means that they are supported by walls on all four sides. This description assumes that a slab is rectangular in plan, as is normally the case.Slabs are usually of reinforced concrete and in buildings they are typically 150–300 millimetres in depth. Larger than normal spans can be achieved by using ribbed or waffle slabs, as shown in Fig. 1.7 (c) and (d). Like beams, slabs experience bending.

Columns

Columns (or 'pillars' or 'posts') are vertical and support axial loads, thus they experience compression. If a column is slender or supports a nonsymmetrical arrangement of beams, it will also experience bending, as shown by the broken line in Fig. 1.8 (a). Concrete ormasonry columns may be of square, rectangular, circular or cruciform cross-section, as illustrated in Fig. 1.8 (b). Steel columns may be H or hollow section.

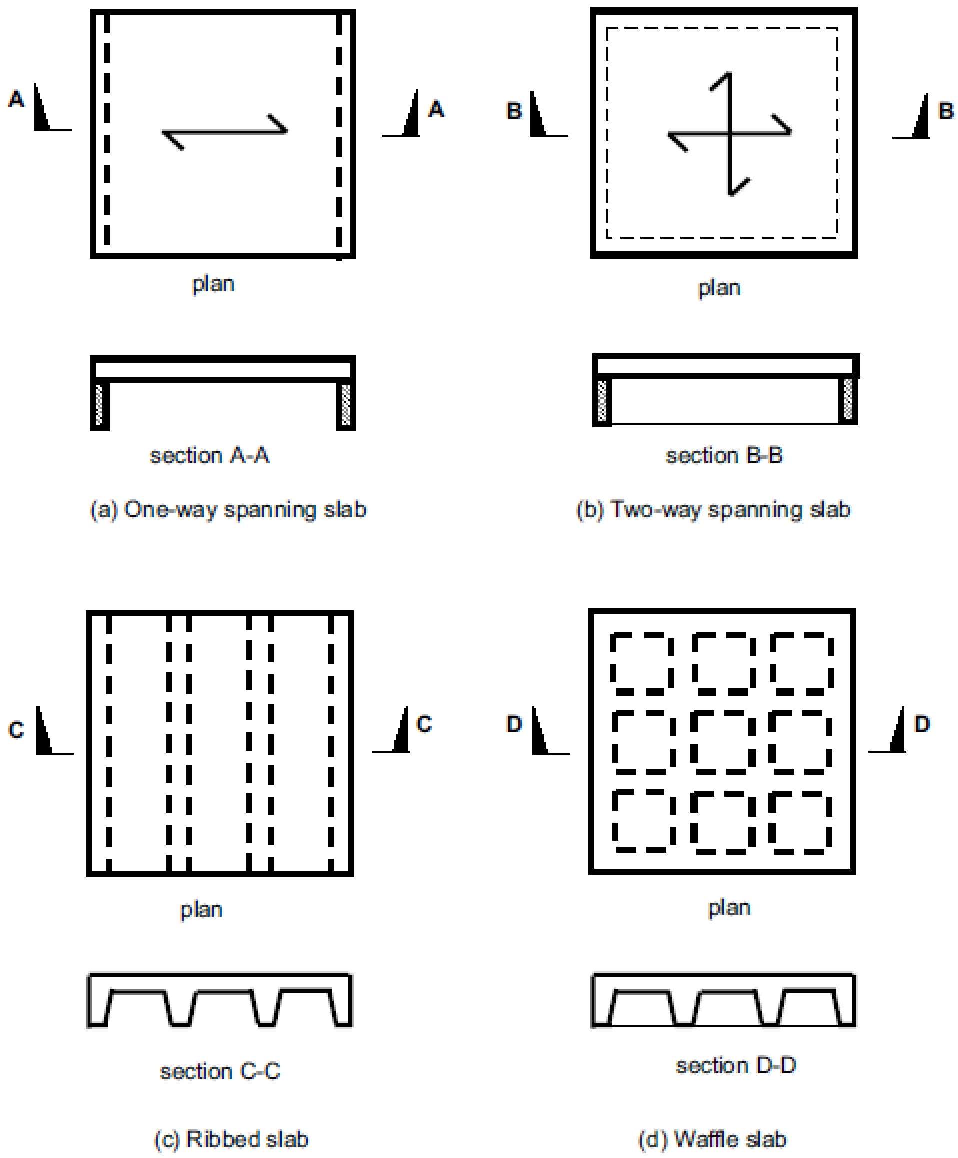


Figure1.7Slabtypes

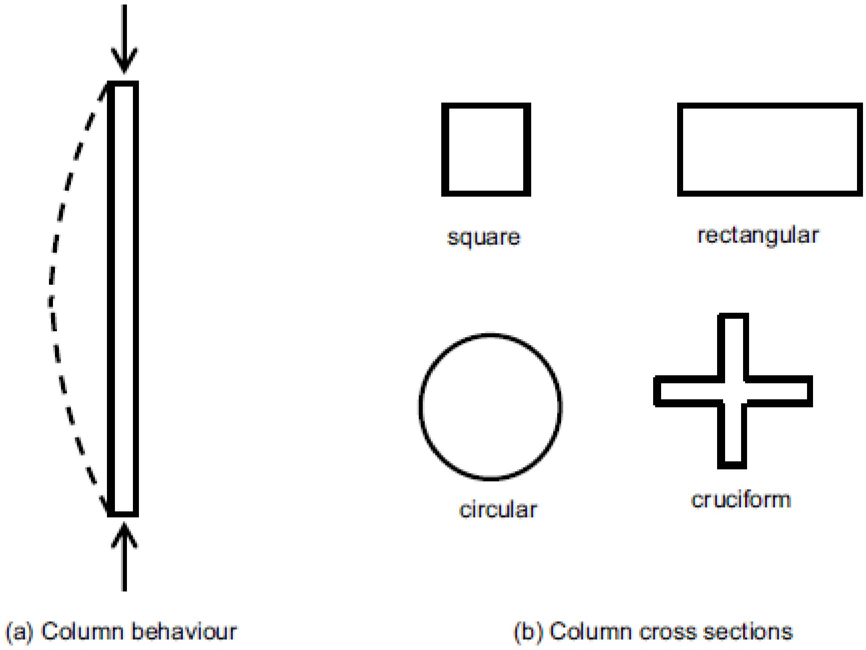


Figure1.8Columntypes

Walls

Like columns, walls are vertical and are primarily subjected to compression, but they may also experience bending. Walls are usually of masonry or reinforced concrete. As well as conventionalflat-facedwallsyoumightencounterfinordiaphragmwalls,asshowninFig. 1.9. Retaining walls hold back earth or water and thus are designed to withstand bending caused by horizontal forces, as indicated by the broken line in Fig. 1.9 (c).

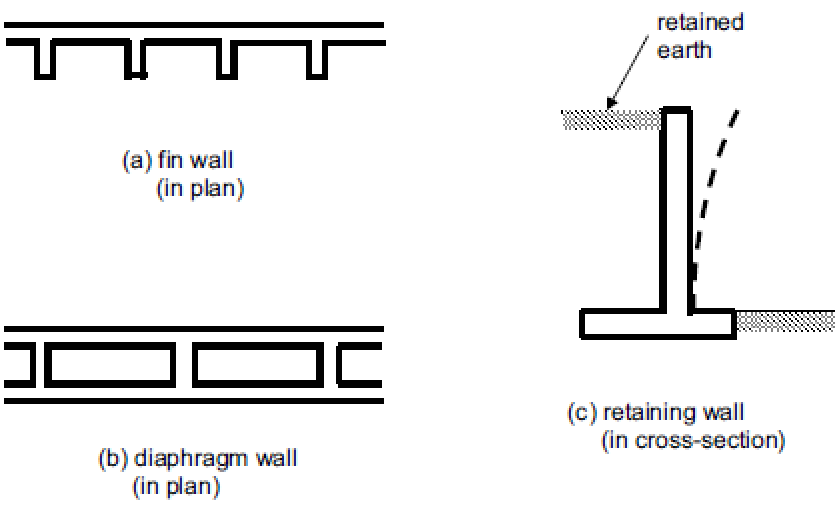


Figure1.9Walltypes

Foundations

As mentioned previously, everything designed by an architect or civil or structural engineer must standontheground–oratleast havesomecontact withtheground.So foundations are required, whose function is to transfer loads from the building safely into the ground. There are various types of foundation. A strip foundation provides a continuous support to load bearing external walls. A pad foundation provides a load-spreading support to a column. A raft foundation takes up the whole plan area under a building and is used in situations where the alternative would be a large number of strip and/or pad foundations in a relatively small space. Where the ground has low strength and/or the building is very heavy, piledfoundations are used.

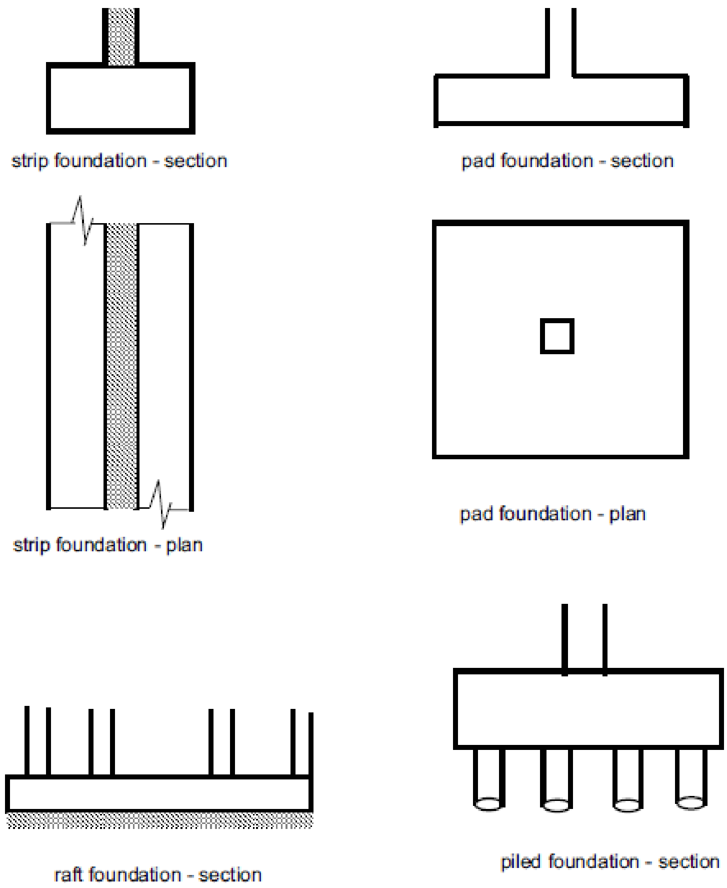


Figure1.10Foundationtypes

These are columns in the ground which transmit the building's loads safely to a stronger stratum. All these foundation types are illustrated in Fig. 1.10. Foundations of all types are usually of concrete, but occasionally steel or timber may be used for piles.

Arches

The main virtue of an arch, from a structural engineering point of view, is that it is in compression throughout. This means that materials that are weak in tension – for example, masonry – may be used to span considerable distances. Arches transmit large horizontal thrusts into their supports, unless horizontal ties are used at the base of the arch. It is to cope with these horizontal thrusts that fl ying buttresses are provided in medieval cathedrals – see Fig. 1.11.

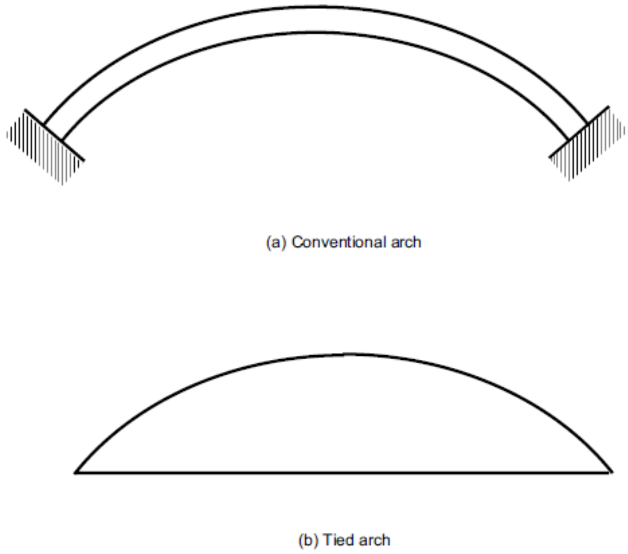


Figure1.11Archtypes

Trusses

A truss is a two- or three-dimensional framework and is designed on the basis that each ‘member’ or component of the framework is in either pure tension or pure compression and does not experience bending. Trusses are often used in pitched roof construction: timbertends to be used for domestic construction and steel caters for the larger roof spans requiredin industrial or commercial buildings. Lattice girders, which are used instead of solid deep beams for long spans, work on the same principle – see Fig. 1.12.

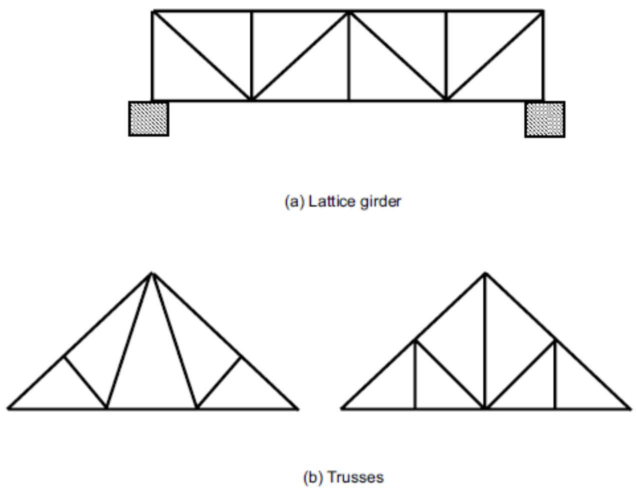


Figure1.12Trusstypes

PortalFrames

A portal frame is a rigid framework comprising two columns supporting rafters. The rafters may be horizontal or, more usually, inclined to support a pitched roof. Portal frames are usually of steel but may be of precast concrete. They are usually used in large single-storey structures such as warehouses or out-of-town retail sheds – see Fig. 1.13.

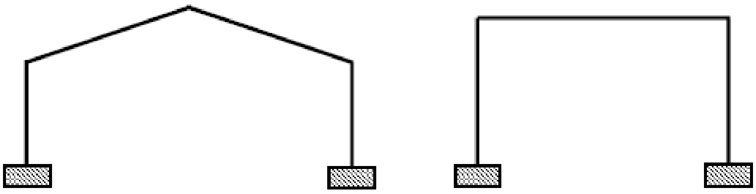


Figure1.13Portalframetypes

Cable stayed and suspension structures

Cable stayed structures are usually bridges but are sometimes used in building structures where exceptionally long spans are required. Instead of being supported from below by columns or walls, the span is supported from above at certain points by cables which pass over supporting vertical masts and horizontal outriggers to a point in the ground where they are firmly anchored. The cables are in tension and must be designed to sustain considerable tensile forces – see Fig. 1.14.

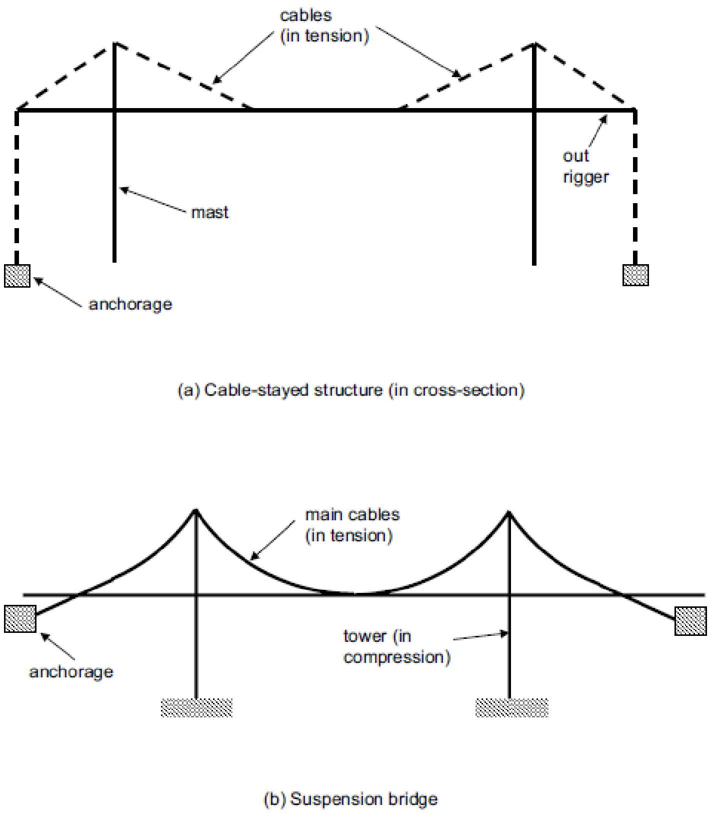


Figure1.14cablestayedandsuspensionstructures

UNIT – V

STIFFNESS METHOD

1. Define static indeterminacy.

The excess number of reactions that make a structure indeterminate is called static indeterminacy.

Static indeterminacy = No. of reactions – Equilibrium conditions

2. Define degree of freedom of the structure with an example.

Degree of freedom is defined as the least no of independent displacements required to define the deformed shape of a structure. There are two types of DOF: (a) Nodal type DOF and (b) Joint type DOF.

3. Write a short note on global stiffness matrices.

The size of the global stiffness matrix (GSM) = No: of nodes x Degrees of freedom per node.

4. List out the properties of rotation matrix.

a) Matrix multiplication has no effect on the zero vectors (the coordinates of the origin).
b) It can be used to describe rotations about the origin of the coordinate system.
c) Rotation matrices provide an algebraic description of such rotations.
d) They are used extensively for computations.
e) Rotation matrices are square matrices with real entries.

5. What are the basic unknowns in stiffness matrix method?

In the stiffness matrix method nodal displacements are treated as the basic unknowns for the solution of indeterminate structures.

6. Define stiffness coefficient 'kij'.

Stiffness coefficient 'kij' is defined as the force developed at joint 'i' due to unit displacement at joint 'j' while all other joints are fixed.

7. What is the basic aim of the stiffness method?

The aim of the stiffness method is to evaluate the values of generalized coordinates 'r' knowing the structure stiffness matrix 'k' and nodal loads 'R' through the structure equilibrium equation.

8. What is the displacement transformation matrix?

The connectivity matrix which relates the internal displacement 'q' and the external displacement 'r' is known as the displacement transformation matrix 'a'.

9. What is meant by generalized coordinates?

For specifying a configuration of a system, a certain minimum no of independent coordinates are necessary. The least no of independent coordinates that are needed to specify the configuration is known as generalized coordinates.

10. Why the stiffness matrix method is also called equilibrium method or displacement method?

Stiffness method is based on the superposition of displacements and hence is also known as the displacement method. And since it leads to the equilibrium equations the method is also known as equilibrium method.

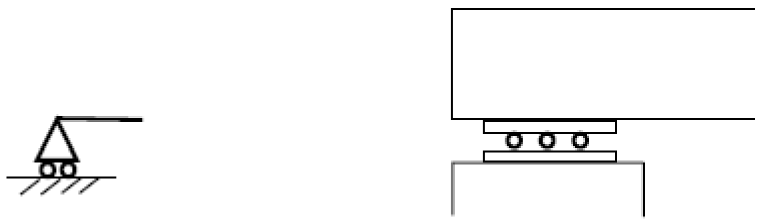
Differenttypesofsupports

Uptillnowwe'vebeentalkingaboutsupports(to beams,etc.) andindicatingthemasupward arrows without giving any thought to the type or nature of the support. As we shall see, there are three different types of support.

RollerSupports

Imagine a person on roller skates standing in the middle of a highly polished floor. If you were to approach this person and give him (or her) a sharp push from behind (not to be recommended without discussing it with them first!), they would move off in the direction you pushed them. Because they are on roller skates on a smooth fl oor, there would be minimal friction to resist the person's slide across the floor.

A roller support to part of a structure is analogous to that person on roller skates: a roller support is free to move horizontally. Roller supports are indicated using the symbol shown in Fig. 2.3 (a). You should recognise that this is purely symbolic and a real roller support will probablynot resemble this symbol. In practice a roller support might comprise sliding rubber bearings, for example, or steel rollers sandwiched between steel plates, as shown in Fig. 2.3 (b).



(a) roller symbol (b) an actual roller support

Figure2.3Rollersymbolicallyandinreality

PinnedSupport

Consider the door hinge analogy discussed above. A pinned support permits rotation but cannot move horizontally or vertically – in exactly the same way as a door hinge provides rotation but cannot itself move from its position in any direction.

FixedSupport

Form your two hands into fists, place them about a foot apart horizontally and allow a friend to position a ruler on your two fists so that it is spanning between them. Your fists are safely supporting the ruler at each end. Now remove one of the supports by moving your fist out from underneath the ruler. What happens? The ruler drops to the floor. Why? You have removed one of the supports and the remaining single support is not capable of supporting theruleronitsown–seeFigs2.4(a)and (b).

However, if you grip the ruler between your thumb and remaining fingers at one end only, it can be held horizontally without collapsing. This is because the firm grip provided by your hand prevents the end of the ruler from rotating and thus fallingto the floor – see Fig. 2.4 (c).

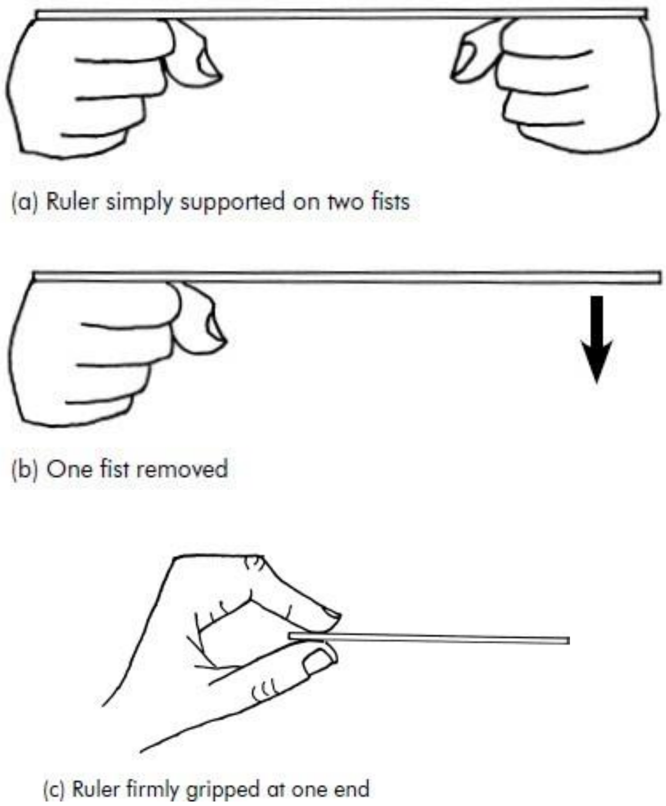


Figure2.4Whatisafixedsupport?

In structures, the support equivalent to your gripping hand in the above example is called afixed support. As with your hand gripping the ruler, a fixed support does not permit rotation. There are many situations in practice where it is necessary (or at least desirable) for a beamorslabtobesupportedatoneendonly–forexample,abalcony.Inthesesituations,the single end support must be a fixed support because, as we’ve seen, a fixed support does notpermit rotation and hence does not lead to collapse of the structural member concerned – see Fig.2.5.Likeapinnedsupport,afixedsupportcannotmoveinanydirectionfromits position. Unlike a pinned support, a fixed support cannot rotate. So a fixed support is fixed in every respect.

Now you’vegotamental pictureofeachofthethreedifferenttypesofsupport(roller,pinned and fixed), let’s revisit each of them and take our studyof them a stage further. We are going to do this in the context of reactions and moments.

Restraints

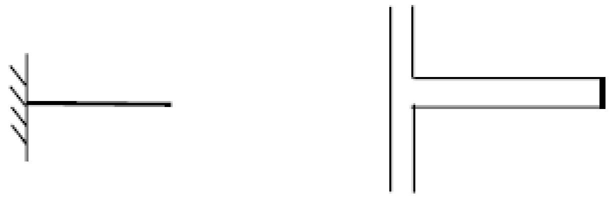
Let’sconsidereachofthefollowingasbeingarestraint:
(1) Verticalreaction
(2) Horizontalreaction
(3) Resistingmoment.

Restraintsexperiencedbydifferenttypesofsupport

Rollersupport

Let's return to our roller skater standing on a highlypolished floor. As the floor is supporting him, it must be providing an upward reaction to counteract the weight of the skater's body. However, we've already seen that if we push our skater, he will move. The rollers on the skates, and the frictionless nature of the floor, mean that the skater can offer no resistance to our push. In other words, the skater can provide no horizontal reaction to our pushing (in contrast to a solid wall, for example, which would not move if leaned on and therefore would provide a horizontal reaction).

There is also nothing to stop the skater from falling over (i.e. rotating). We can see from the above that a roller support provides one restraint only: vertical reaction. (There is no horizontal reaction and no moment.)



(a) fixed support symbol (b) an actual fixed support

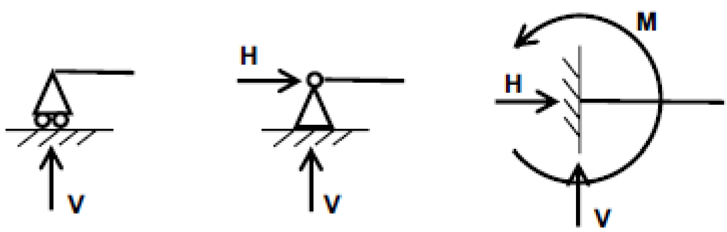
Figure2.5Fixedsupportsymbolicallyandinreality

Pinnedsupport

As discussed above, a pinned support permits rotation (so there is no resistance to moment), but as it cannot move horizontally or vertically there must be both horizontal and vertical reactions present. So, a pinned support provides two restraints: vertical reaction and horizontal reaction. (There is no moment.)

FixedSupport

We saw above that a fixed support is fixed in every respect: it cannot move eitherhorizontally or vertically and it cannot rotate. This means there will be both horizontal and vertical reactions and, if it cannot rotate, there must be a moment associated with the fixed support. Incidentally, this moment is called a fixed end moment. So, a fixed support provides three restraints: vertical reaction, horizontal reaction and moment.



(a) roller support (b) pinned support (c) fixed support

Figure2.6Restraintsprovidedbyvarioussupporttypes.

Tosummarise:
• Arollersupportprovidesonerestraint:verticalreaction.
• Apinnedsupportprovidestworestraints:verticalreactionandhorizontalreaction.
• Afixedsupportprovidesthreerestraints:verticalreaction,horizontalreactionandmoment.

ThisisillustratedinFig.2.6.

SolutionofEquilibriumEquations

Weknowfromourknowledgeofmathematicsaboutthefollowing:
• If we have the same number of unknowns as we have equations, a mathematical problemcan be solved.
• Butifwehavemoreunknownsthanequations,amathematicalproblemcannotbesolved.

Relating this to structural analysis, if we look back to the procedure we used for calculating reactions, we'll see that we were solving three equations. These equations were represented by:

(1) Verticalequilibrium(totalforceup=totalforcedown)
(2) Horizontalequilibrium(totalforceright=totalforceleft)
(3) Momentequilibrium(totalclockwisemoment=totalanticlockwisemoment).

StaticallyDeterminatevsStaticallyindeterminate

As we have three equations, we can use them to solve a problem with up to three unknownsin it. In this context, an unknown is represented bya restraint, as defined earlier. (Remember, a roller support has one restraint, a pinned support has two restraints and a fixed support has three restraints.)Once these reactions are evaluated, we could determine the internal stress resultants (reactions, axial forces, moments etc.) in the structure. Correct solution for reaction and internal stresses must satisfy the equations of static equilibrium for the entire structure. They must also satisfy equilibrium equations for any part of the structure as a free body. A sketch depicting the free body with the associated forces and internal stresses is called a **free body diagram** (FBD). Hence a structural system with up to three restraints is solvable – such a system is said to be statically determinate (SD) – while a structural system with more than three restraints is not solvable (unless we use advanced structural techniques) – such a system is said to be statically indeterminate (SI).

So if we inspect a simple structure, examine its support and thence count up the number of restraints, we can determine whether the structure is statically determinate (up to three restraints in total) or statically indeterminate (more than three restraints).

Let'slookatthethreeexamplesshowninFig.2.7.

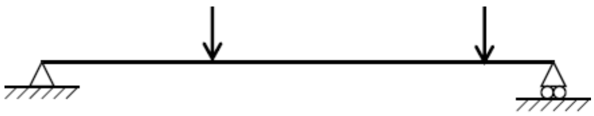
Example 1
This beam has a pinned support (two restraints) at its left-hand end and a roller support (one restraint) at its right-hand end. So the total number of restraints is (2 + 1) = 3, therefore the problem is solvable and is statically determinate (SD).

Example2

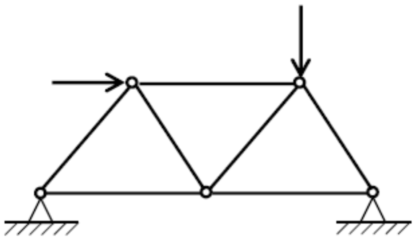
This pin-jointed frame has a pinned support (two restraints) at each end. So the total number of restraints is (2 + 2) = 4. As 4 is greater than 3, the problem is not solvable and is statically indeterminate (SI).

Example3

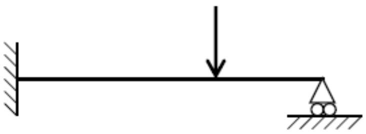
This beam has a fixed support (three restraints) at its left-hand end and a roller support (one restraint) at its right-hand end. So the total number of restraints is (3 + 1) = 4, therefore,again, the problem is not solvable and is statically indeterminate (SI).



(a) Example 1



(b) Example 2



(c) Example 3

Figure2.7Staticaldeterminancy

SampleProblems

Problem 1:

Determine whether each of the structures given in Fig. 2.8 is statically determinate (SD) orstatically indeterminate (SI).

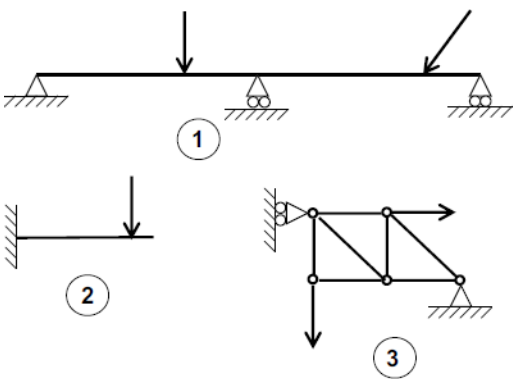


Figure2.8StaticaldeterminancySampleproblems

Stability

It is essential for a structure to be strong enough to be able to carrythe loads and moments to which it will be subjected. But strength is not sufficient: the structure must also be stable.

Stabilityofstructuralframeworks

Many buildings and other structures have a structural frame. Steel buildings comprise a framework, or skeleton, of steel. We are going to consider the build-up of a framework from scratch. Our framework will consist of metal rods (‘members’) joined together at their endsby pins. (The concept of a pin, which is a type of connection that facilitates rotation)Consider two members connected by a pin joint, as shown in Fig. 3.1 (a). Is this a stable structure? (In other words, is it possible for the two members to move relative to each other?) As thepin allows thetwomembers to moverelativeto oneanother, this is clearlynot astable structure.

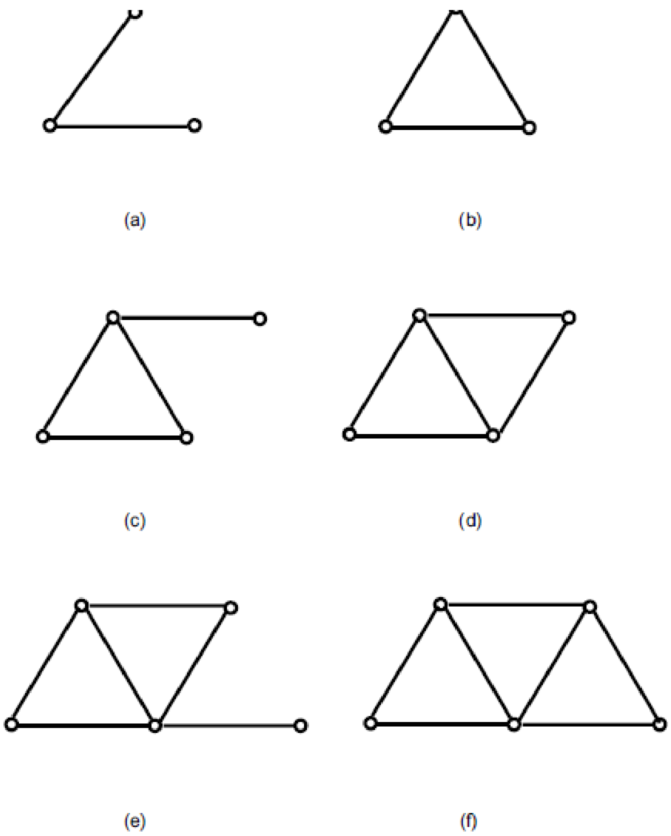


Figure3.1Buildingupaframework

Now, let’s add a third member to obtain three members connected by pin joints to form a triangle, as shown in Fig. 3.1 (b). Is this a stable structure? Yes, it is because even though the joints are pinned, movement of the three members relative to each other is not possible. So this is a stable, rigid structure. In fact, the triangle is the most basic stable structure, as wewillmentionagaininthefollowingdiscussion.Ifweaddafourthmemberweproducethe frame shown in Fig. 3.1 (c). Is this a stable structure? No it is not. Even though the triangle within it is stable, the 'spur' member is free to rotate relative to the triangle, so overall this is not a stable structure. Consider the frame shown in Fig. 3.1 (d), which is achieved by addinga fifth member to the previous frame. This is a stable structure. Ifyou are unsure of this, trytodeterminewhichindividualmember(s)withintheframecanmoverelativetotherestofthe frame. You should see that none of them can and therefore this is a stable structure. This is why you often see this detail in structural frames as 'diagonal bracing', which helps to ensure the overall stability of a structure.

Let's add yet another member to obtain the frame shown in Fig. 3.1 (e). Is this a stable structure? No, it is not. In a similar manner to the frame depicted in Fig. 3.1 (c), it has a spur member which is free to rotate relative to the rest of the structure. Adding a further member we can obtain the frame shown in Fig. 3.1 (f) and we will see that this is a rigid, or stable, structure.

We could carry on ad infinitum in this vein, but I think you can see that a certain pattern is emerging. The most basic stable structure is a triangle (Fig. 3.1 (b)). We can add two members to a triangle to obtain a 'new' triangle. All of the frames that comprise a series of triangles (Figs 3.1 (d) and (f)) are stable; the remaining ones, which have spur members, are not.

Let's now see whether we can devise a means of predicting mathematically whether a given frame is stable or not. In Table 3.1 each of the six frames considered in Fig. 3.1 is assessed. The letter m represents the number of members in the frame and j represents the number of joints(notethatunconnectedfreeendsofmembersarealsoconsideredasjoints).Thecolumn headed 'Stable structure?' merely records whether the frame is stable ('Yes') or not ('No').

Table 3.1 Is a structure stable?

	m	*j*	Stable structure?	$2j-3$	**Is $m = 2j - 3$?**
Figure 3.1 (a)	2	3	No	3	No
Figure 3.1 (b)	3	3	Yes	3	Yes
Figure 3.1 (c)	4	4	No	5	No
Figure 3.1 (d)	5	4	Yes	5	Yes
Figure 3.1 (e)	6	5	No	7	No
Figure 3.1 (f)	7	5	Yes	7	Yes

Itcanbeshownthatif$m = 2j - 3$thenthestructureisstable. Ifthatequationdoesnothold, then the structure is not stable. This is borne out by Table 3.1: compare the entries in the column headed 'Stable structure?' with those in the column headed 'Is $m = 2j - 3$?'.

Internalstabilityofaframedstructure-asummary

(1) A framework which contains exactly the correct number of members required to keep it stable is termed a ***perfect frame***. In these cases, $m = 2j - 3$, where *m* is the number ofmembersintheframeand*j*isthenumberofjoints(includingfreeends).Frames(b),(d)and (f)inFig.3.1areexamples.

(2) A framework having less than the required number of members is unstable and is termeda ***mechanism***. In these cases, $m < 2j - 3$. Frames (a), (c) and (e) in Fig. 3.1 are examples. In each case, one member of the frame is free to move relative to the others.

(3) A framework having more than this required number is 'over-stable' and contains redundant members that could (in theory at least) be removed. Examples follow. In these cases, $m > 2j - 3$. These frames are staticallyindeterminate (SI) whichmeans that the frames cannot be mathematically analysed without resorting to advanced structural techniques.

Examples

For each of the frames shown in Fig. 3.2, use the equation $m = 2j - 3$ to determine whether the frame is (a) a perfect frame (SD), (b) a mechanism (Mech) or (c) staticallyindeterminate (SI).Wherethe frameisamechanism,indicatethemannerinwhichtheframecoulddeform. Where the frame is statically indeterminate, consider which members could be re-

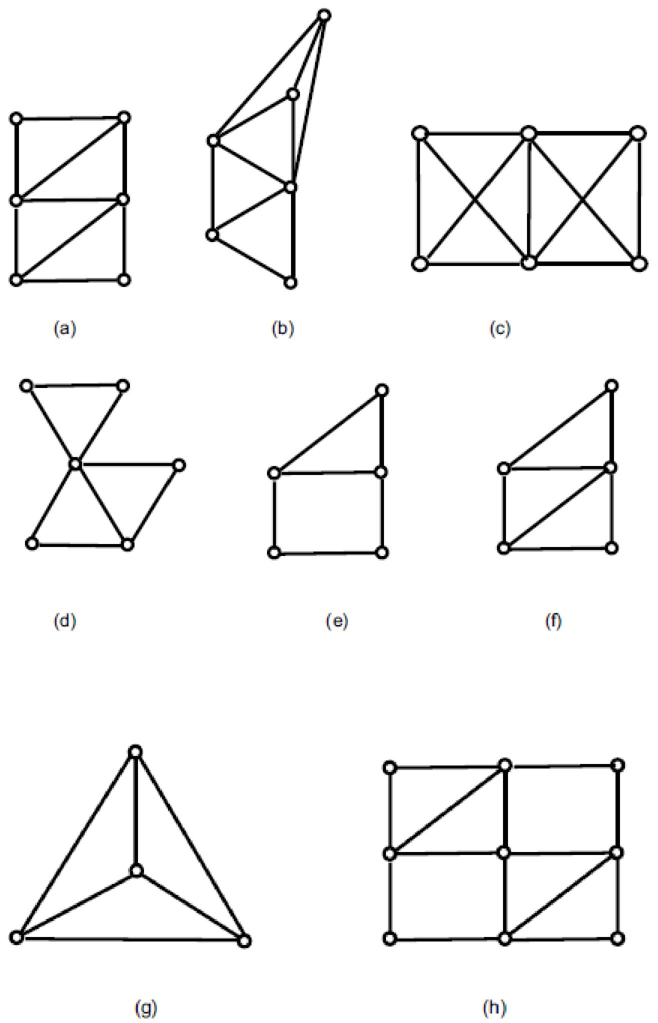


Figure3.2Aretheseframesstable?

moved without affecting the stability of the structure. The answers are given in Table 3.2.TheframesshowninFigs3.2(b),(c)and(g)arestatically indeterminate.Thismeansthey areover-stableandthatoneormoremembersmayberemovedwithoutcompromising stability. In the case of Fig. 3.2 (b), anyone member can be removed from the top part of the frame and the structure would still be stable. In Fig. 3.2 (c), two members could be removed without compromising stability – but the two members to be removed should be chosen with care.Asensiblechoicewouldbetoremoveonediagonalmemberfromeachofthetwo squares.InFig.3.2(g),any onemembercouldberemoved.TheframesshowninFigs3.2 (d), (e) and (h) are mechanisms. This means that a part of the frame is able to move relativetoanotherpartoftheframe.InFig.3.2(d),theuppertriangleisfreetorotateaboutthe frame'scentralpinindependently ofthe lower partoftheframe.InFig.3.2(e),thesquare part of the frame is free to deform, or collapse, as we shall see in a later example. The mode of deformation of the frame in Fig. 3.2(h) is less easy to visualise. It is shown in Fig. 3.3.

Table 3.2 Stability of frames shown in Fig. 3.2

	m	*j*	2*j* – 3	Is *m* = 2*j* – 3? (or > or <)	**Stability type**
Figure 3.2 (a)	9	6	9	=	SD
Figure 3.2 (b)	10	6	9	>	SI
Figure 3.2 (c)	11	6	9	>	SI
Figure 3.2 (d)	8	6	9	<	Mech
Figure 3.2 (e)	6	5	7	<	Mech
Figure 3.2 (f)	7	5	7	=	SD
Figure 3.2 (g)	6	4	5	>	SI
Figure 3.2 (h)	14	9	15	<	Mech

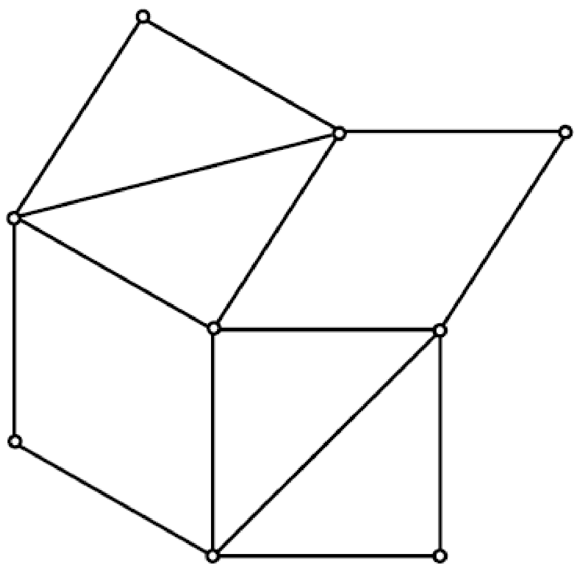
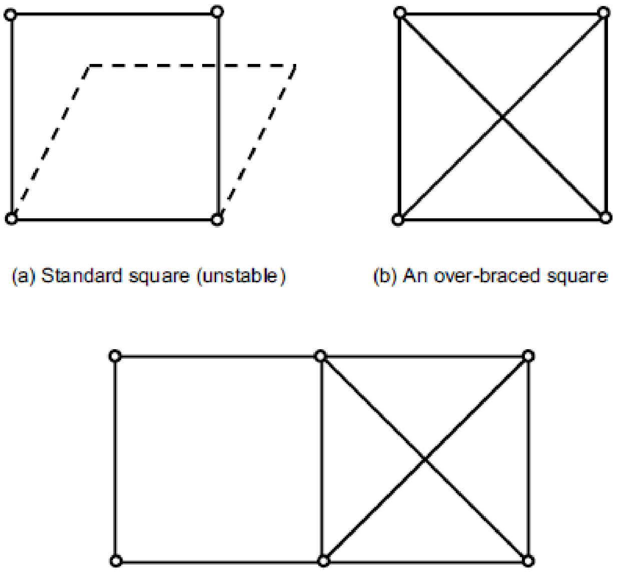


Figure3.3DeformationofframeshowninFigure3.2(h)

Generalcases

Look at frames (a) and (b) in Fig. 3.4. If we apply the $m = 2j - 3$ formula to the standard square depicted in Fig. 3.4 (a), we will find that it is unstable, or a mechanism. It can deform in the manner indicated by the broken lines in Fig. 3.4 (a). This is why, in 'real' structures, diagonal cross-bracing must often be provided to ensure stability.

If we look at the frame shown in Fig. 3.4 (b), we see that it is a square which is diagonally cross-braced twice. Applyingthe$m$ =2j–3 formulawefindthatitisstaticallyindeterminate, which means that it contains at least one redundant member. On further investigation we find that we can remove any one of the six members without affecting the stability of thestructure.



(a) Standard square (unstable) (b) An over-braced square

(c) Perfect frame, mechanism or over-braced?

Figure3.4Framestabilitygeneralcases

Consider the frame showninFig.3.4(c).Itcontainsnine membersandsixjoints,so$m$= 9 and j = 6 and it can thus readilybe shown that $m = 2j – 3$ in this case, which suggests that the frameworkis aperfectframe. In fact,an inspectionoftheframeshows that thisisnot,infact, the case. The left hand part of the frame is an un-braced square, which is a mechanism and can deform in the same manner as the frame shown in Fig. 3.4 (a). But the right-hand part of theframehasdoublediagonalcross-bracing,whichsuggeststhatitis'over-stable'and contains redundant members in the same way as the frame shown in Fig. 3.4 (b). So, part of the frame shown in Fig. 3.4 (c) is a mechanism and the other part is statically indeterminate, but this does not make an overall perfect frame, as predicted bythe formula! The lesson to be learned fromthisisthattheformula$m$ =2j–3shouldberegarded asaguideonly– itdoesn't always work. A given frame should always be inspected to see whether there are anysigns of either (a) mechanism or (b) over-stability.

FramesonSupports

Up till now we have conveniently ignored the fact that, in practice, frames have to be supported. We therefore need to consider the effects of supports on the overall stability of frames. weknew about thethree different types ofsupport (roller, pinned and fixed). Wealso saw that:

- arollersupportprovidesonerestraint(r=1);
- apinnedsupportprovidestworestraints(r=2);
- afixedsupportprovidesthreerestraints(r=3).

The $m = 2j - 3$ used above is now modified to $m + r = 2j$ where supports are present. As before, m is the number of members and j is the number of joints. The letter r represents the total number of restraints (one for each roller support, two for each pinned support and three for each fixed support).

(1). If $m + r = 2j$, then the frame is a perfect frame and is statically determinate (SD), which means it can be analysed by various methods.

(2) If $m + r < 2j$, thentheframeisamechanism– itisunstableandshouldnotbeusedasa structure.

(3) If $m + r > 2j$, then the frame contains redundant members and is staticallyindeterminate (SI),whichmeansitcannotbeanalysedwithoutresortingtoadvancedmethodsofstructural analysis.

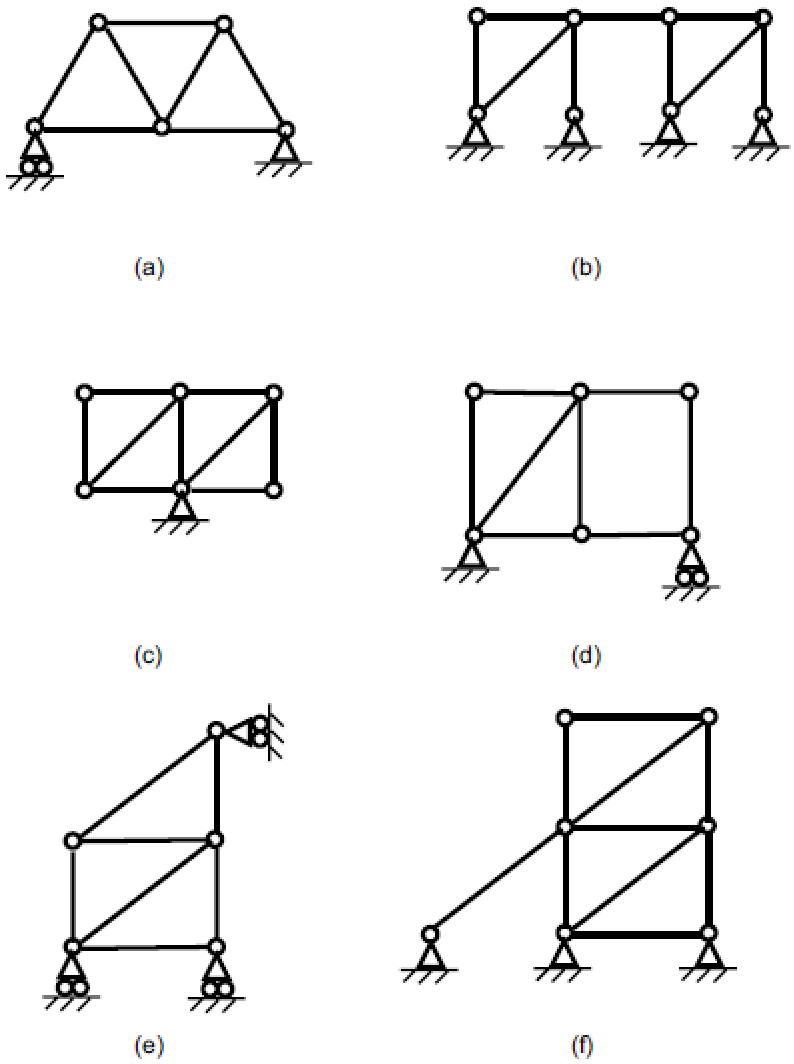


Figure3.5Arethesestructuresstable?

Examples

For eachof the frames shown in Fig. 3.5, use the equation $m + r = 2j$ todetermine whethertheframeis(a) staticallydeterminate,(b)amechanismor(c)staticallyindeterminate.Where the frame is a mechanism, indicate the manner in which the frame could deform. Where the frame is statically determinate, consider which members could be removed without affecting the stability of the structure. The answers are given in Table 3.3.

Table 3.3 Stability of structures shown in Fig. 3.5

	m	*j*	2*j*	*r*	*m* + *r*	Is *m* + *r* = 2*j*? (or > or <)	**Stability type**
Figure 3.5 (a)	7	5	10	3	10	=	SD
Figure 3.5 (b)	9	8	16	8	17	>	SI
Figure 3.5 (c)	9	6	12	2	11	<	Mech
Figure 3.5 (d)	8	6	12	3	11	<	Mech
Figure 3.5 (e)	7	5	10	3	10	=	SD
Figure 3.5 (f)	10	7	14	6	16	>	SI

The frames shown in Figs 3.5 (b) and (f) are statically indeterminate. This means they are over-stable and that one ormoremembers mayberemoved. In thecaseof Fig. 3.5 (b), oneof the diagonal members may be removed (but not both of them!) and the structure would stillbe stable. In Fig. 3.5 (f), the 'lean-to' diagonal member may be removed without compromising stability. The frames shown in Figs 3.5 (c) and (d) are mechanisms. The structure in Fig. 3.5 (c) is obviously unstable, being free to rotate about its single central support. In Fig. 3.5 (d), the square part of the frame is free to deform in the manner indicated in Fig. 3.4 (a).

Stabilityofrealstructures

Inpractice,thestabilityofastructureisassuredinoneofthreeways:

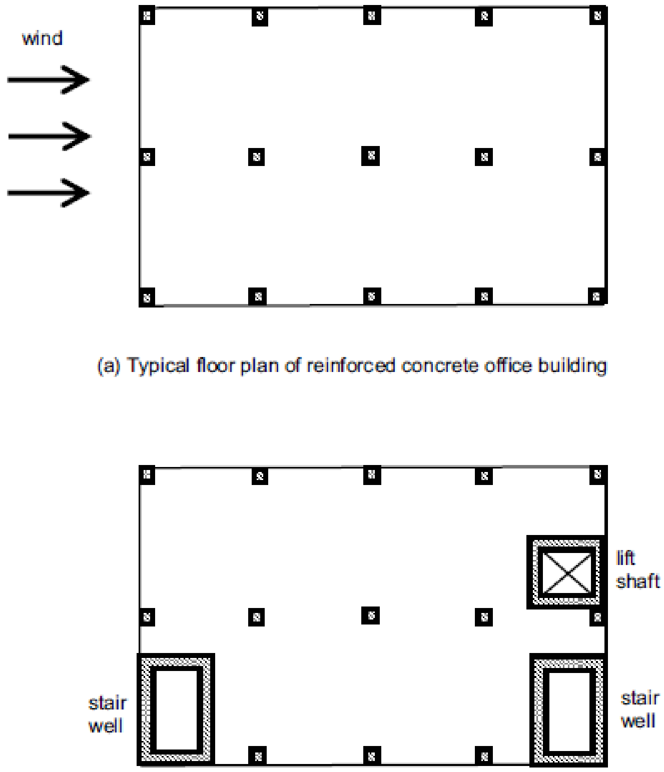
(1) Shearwalls/stiffcore.
(2) Cross-bracing.
(3) Rigidjoints.

Let'slookateachoftheseinmoredetail.

Shearwalls/stiffcore

This form of stabilityis usually(but not exclusively) used in concrete buildings. Consider the structural plan ofan upperfloorofatypical concreteofficebuilding, as shown in Fig. 3.6 (a). The structure comprises a grid layout of columns, which support beams and slabs at each floor level. The wind blows horizontally against the building from any direction. It is obviously important that the building doesn't collapse in the manner of a 'house of cards' under the effects of this horizontal wind force. We could design each individual column to resist the wind forces, but for various reasons this is not the way it is normally done.

Instead, shear walls are used. These walls are designed to be stiff and strong enough to resist all the lateral forces on the building. Since most buildings have staircases and many have lift shafts, the walls that surround the staircases and lift shafts are often designed and constructed toperformthisrole,asshowninFig.3.6(b).Onlargerbuildings,theshearwallsmaybe constructed in such a way as to comprise an inner core to the building, which often contains stairwells, lift shafts, toilets and ducts for services.



(a) Typical floor plan of reinforced concrete office building

(b) Same floor plan with shear walls added

Figure3.6Provisionofstabilityusingshearwalls

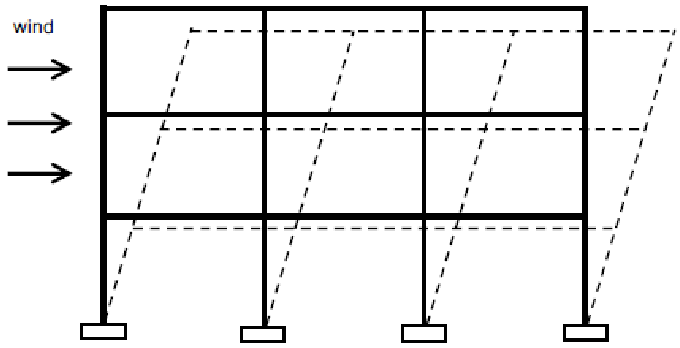
Crossbracing

Thisformofstabilityis commoninsteel-framedbuildings.Figure3.7 (a)showstheelevation of a three-storey steel-framed building, on which the wind is blowing. There is nothing tostop the building tilting over and collapsing in the manner indicated by the broken lines in Fig. 3.7 (a).

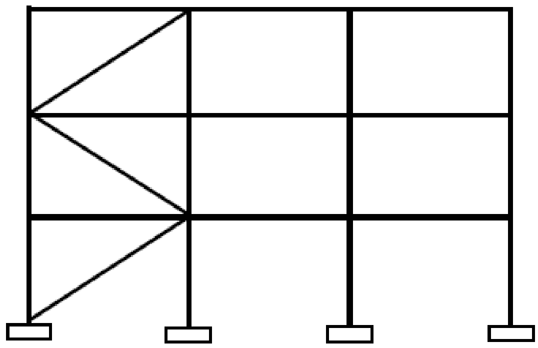
One wayof ensuringstabilityis to stop the ‘squares’ in the buildingelevation from becoming trapeziums. Earlier in this chapter we saw that (a) a triangle is the most basic stable structure and(b)adiagonalmembercanstopasquarefromdeforming(illustratedinFigs3.1(b)and (d) respectively). So diagonal cross-bracing is used to ensure stability, as shown in Fig. 3.7 (b).

RigidJoints

A third method of providing lateral stability is simply to make the joints strong and stiff enough that movement of the beams relative to the columns is not possible. The black blobs in Fig. 3.8 indicate stiff joints that stop the action depicted in Fig. 3.7 (a) from happening.



(a) Section through three-storey steel framed building



(b) Same section with diagonal bracing added

Figure3.7Provisionofstabilityusingcrossbracing

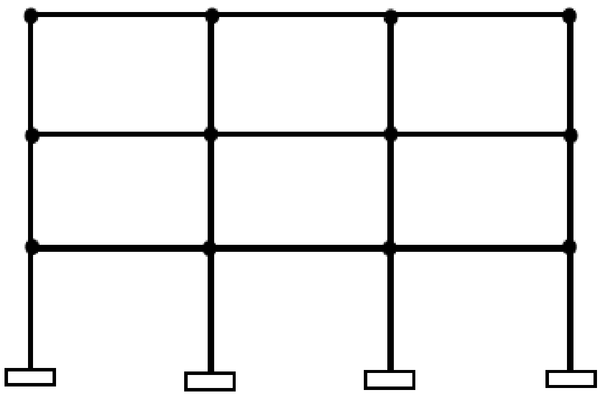


Figure3.8Provisionofstabilityusingrigidframes

SpaceFrames

Normally the centre lines of bars, forces applied and support reactions in the case of plane trusses lie in a plane. When all these lie in different planes i.e in three dimensional space,such a structure is called a space truss or space frame which is nothing but an assemblage of bars in three dimensional space. Tetrahedron is the simplest space frame consisting of six members. Antenna towers, transmission line towers, guyed masts, derricks, offshore structures etc are some of the common examples of space frames. We can construct a space framefromthebasictetrahedronbyaddingthreenewmembersandajoint.Togetastable space frame, we have to arrange adequate number of bars in a suitable manner starting with a basic tetrahedron. There are six bars and four joints in the basic tetrahedron. For each joint added, we have now three additional members. Therefore, we can have a reaction between the member of bars (b) and the number of joints (j) as given below

b-6=3(j-4)

b=3j-6

The above expression gives the minimum number of bars required to construct a stable space truss or space frame. If the number of bars in the space truss is less than that required by the above expression,then we consider the space frame as unstable. In Contrast,if the number of bars is more than the minimum number required then the space frame is considered internally indeterminate.

Sample Problems

Problem 1:

(1) For each of the examples shown in Fig. 3.9, determine whether the frame is (a) a perfect frame(SD),(b)unstable(amechanism)or(c)over-stable(containingredundantmembers). If the framework is unstable, state where a member could be added to make it stable. If the frame is over-stable, determine which members could be removed and the structure would still be stable.

(2) Select a framed structure near where you live. Determine how lateral stability is provided to the structure and state the reasons why the designer may have chosen that particular method of ensuring stability.

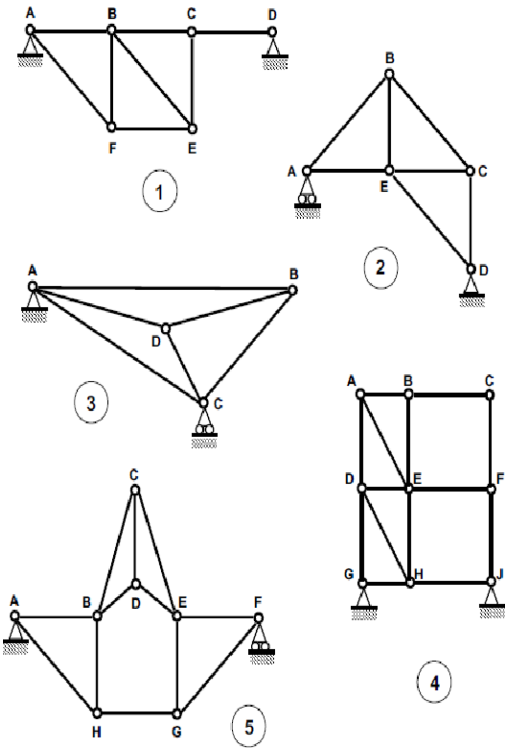


Figure3.9Sampleproblems

DegreesofFreedom

The degrees of freedom (DOF) can be defined as asset of independent displacements that specify completely the deformed position and orientation of the body or system under loading. Hence, displacements include deflections and rotations as well. A rigid body that moves in 3D space in linear directions has three translational displacement components as DOFs. The rigid bodycan also undergo angular motion, which is called rotation. So the body has three rotational DOFs. Altogether a rigid body can have at least six DOFs, three translations and three rotations. Translation refers to the ability of a body to move without rotating whereas rotation refers to its angular motion about some axis. When a structure is loaded, the joints also called nodes will undergo unknown displacements. These displacements are referred to as the DOF for the structures.

Kinematics

We have discussed in the previous sections that loads applied on structural systems in turn induce internal forces in the system. As a consequence of this the system undergoes deformation which generically is called as motion. The study relating to forces and motions constitutes an applied science which is a branch of mechanics. The cardinal principle underlying this bodyis the equilibrium. It is a condition which describes a state of balance of a system when forces applied on it. As the structural system is initially at rest and in equilibriumtoounderasystemofforces actingin it,wecallthatpartofmechanics concerned with relations between these forces as **statics**. There is another part of mechanics called **dynamics** which refers to the other part of mechanics dealing with rigid bodies of motion. Dynamicsisdividedintotwoparts,namely,**kinematics** and **kinetics**.Kinematicsisthestudy of the geometry of motion. It is used to relate displacement, velocity, acceleration and time without any reference to the forces causing the motion. **Kinetics**is the study of the relation existing between the forces acting on a body, the mass of the body and the motion of the body. It is used to predict the motion caused by the given forces or to determine the forces required to produce a given motion.

In structural analysis, kinematics refers to quantities associated with geometry, the position changes or the deformation of the geometry. This term is used in opposition to the term statics.

Displacement refers to a translation or a rotation of a specific point in a structure. For example, we consider a simple beam as shown in Figure 4.1.

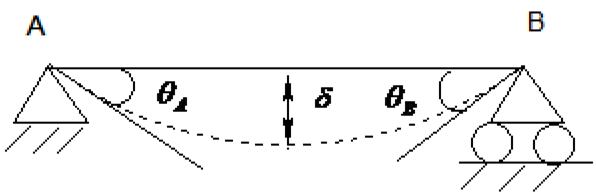


Figure4.1Displacementofsimplebeam

It is free to undergo displacement in the form of translation in the direction perpendicular to itsownaxisasshowninFigure4.1. which is called deflection as well as rotate at its supports. Thequantityδistheverticaltranslationofthebeamandiscalleddeflectionofthebeam. The Rotation atsupportAis$\theta_A$andatsupportBisθ_B.Theserotationsarecalledslopes.

AjointinatrusscantranslateintwomutuallyperpendiculardirectionsasshowninFigure 4.2.ThejointCcandisplacealongxandydirectionsonly.Thejointcannotrotate.

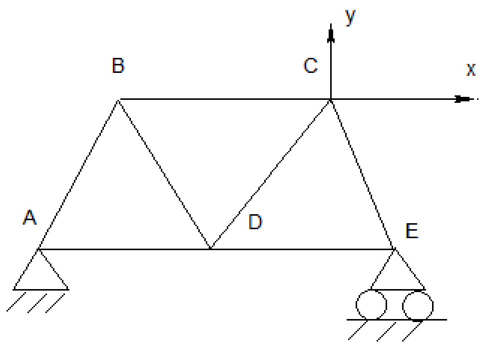


Figure4.2Displacementinatruss

ArigidframecanundergotranslationandrotationatjointsasshowninFigure4.3.Thejoint BinFigure4.3undergoeshorizontaltranslation$\Delta_B$andarotationθ_B.

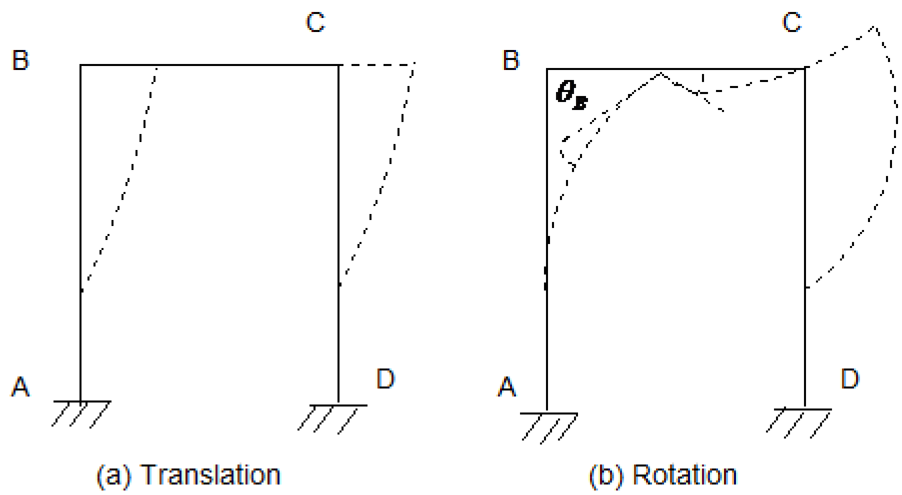


Figure4.3Displacementinaframe

These translations and rotations constitute the degrees of freedom of a structural system. In structural analysis, these displacements other than that at the supports are in general not known. Therefore, the objective of the analysis is to determine their values. The number of the independent joint displacement in a structure is called the degree of kinematic indeterminancy or the number of degrees of freedom. This number is a sum of the degree of freedom in rotation and translation. For example, in a two span beam as shown in Figure 44, thedegreeof kinematicindeterminancyis 2 since thestructure can undergo rotations at joints BandCandthesearei ndeterminates.RotationD1atjointBandrotationD2atjointCare thetwounknowns.BecausesupportAisfixed,therotationD3iszerowhichisaknown quantity and hence determinate.

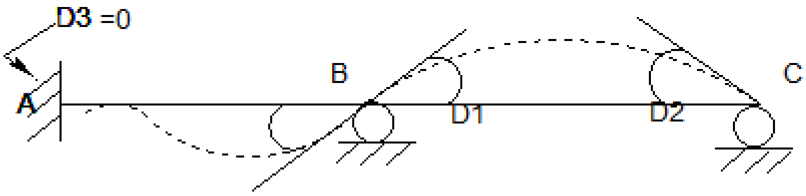


Figure4.4:Degreeofkinematicindeterminancy

5.1. Introduction

Beams are usually straight horizontal members used primarily to carry vertical loads. Quite often they are classified according to the way they are supported, as indicated in Fig. 5.1. In particular, when the cross section varies the beam is referred to as tapered or hunched. Beam cross sections may also be "built up" by adding plates to their top and bottom.

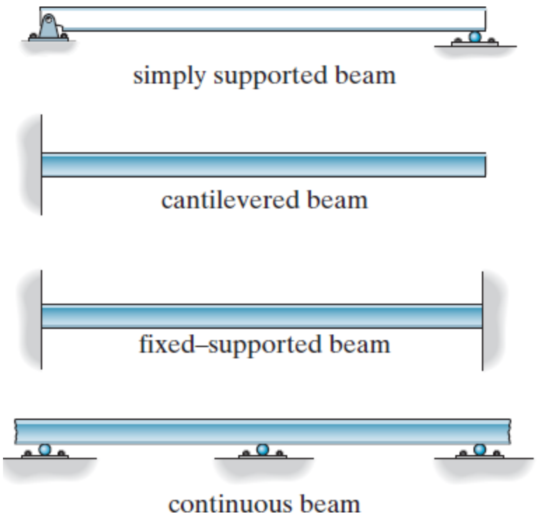


Figure5.1Typesofbeams

Beams are primarily designed to resist bending moment; however, if they are short and carry large loads, the internal shear force may become quite large and this force may govern their design. When the material used for a beam is a metal such as steel or aluminium, the cross section is most efficient when it is shaped as shown in Fig. 5.2. Here the forces developed in the top and bottom flanges of the beam form the necessary couple used to resist the applied moment M, whereas theweb is effectivein resistingtheapplied shearV. This cross section is commonly referred to as a "wide flange," and it is normally formed as a single unit in a rolling mill in lengths up to 23 m. If shorter lengths are needed, a cross section havingtapered flanges is sometimes selected. When the beam is required to have a very large span and the loads applied are rather large, the cross section may take the form of a plate girder. This member is fabricated by using a large plate for the web and welding or bolting plates to its ends for flanges. The girder is often transported to the field in segments, and the segments aredesigned to bespliced orjoined togetherat points wherethegirdercarries asmall internal moment.

Concrete beams generally have rectangular cross sections, since it is easy to construct this form directly in the field. Because concrete is rather weak in resisting tension, steel "reinforcing rods" are cast into the beam within regions of the cross section subjected to tension.Precastconcretebeamsorgirdersarefabricatedatashopor yardinthesamemanner and then transported to thejob site. Beams made from timbermaybesawn from asolid piece of wood or laminated. Laminated beams are constructed from solid sections of wood, which are fastened together using high-strength glues.

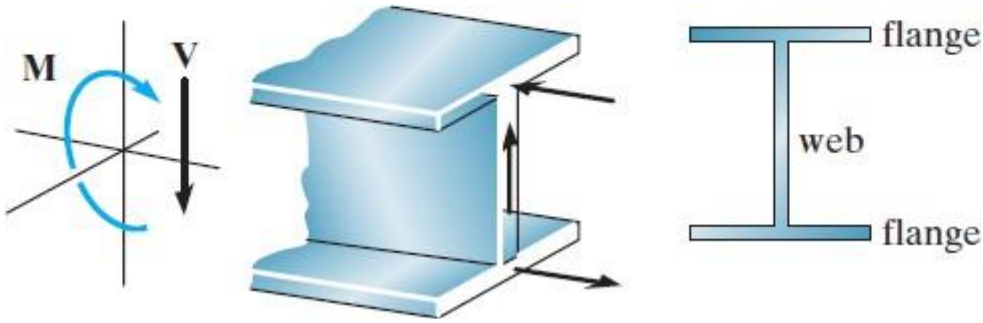


Figure5.2Crosssectionofabeam

Loads

Oncethedimensionalrequirementsforastructurehavebeendefined,itbecomesnecessaryto determine the loads the structure must support. Often, it is the anticipation of the variousloads that will be imposed on the structure that provides the basic type of structure that willbe chosen for design. For example, high-rise structures must endure large lateral loadings caused bywind, and so shearwalls and tubularframesystems areselected, whereas buildings located in areas prone to earthquakes must be designed having ductile frames andconnections.

Once the structural form has been determined, the actual design begins with those elementsthataresubjectedtotheprimaryloadsthestructureisintendedtocarry,andproceedsin sequence to the various supporting members until the foundation is reached. Thus, a building floor slab would be designed first, followed by the supporting beams, columns, and last, the foundation footings. In order to design a structure, it is therefore necessary to first specifythe loads that act on it.

Since a structure is generally subjected to several types of loads, a brief discussion of theseloadings will now be presented to illustrate how one must consider their effects in practice.

Deadload

Dead loads consist of the weights of the various structural members and the weights of any objects that are permanently attached to the structure. Hence, for a building, the dead loads include the weights of the columns, beams, and girders, the floor slab, roofing, walls, windows, plumbing, electrical fixtures, and other miscellaneous attachments.

In some cases, a structural dead load can be estimated satisfactorily from simple formulas based on the weights and sizes of similar structures. Through experience one can also derivea “feeling” for the magnitude of these loadings.

For example, the average weight for timber buildings is 1.9-2.4 KN/m^2 for steel framed buildings it is 2.9-3.6 KN/m^2 and for reinforced concrete buildings it is 5.3-6.2 KN/m^2 Ordinarily, though, once the materials and sizes of the various components of the structureare determined, their weights can be found from tables that list their densities.

Although calculation of dead loads based on the use of tabulated data is rather straightforward, it should be realized that in many respects these loads will have to be estimated in the initial phase of design. These estimates include non-structural materials such as prefabricated facade panels, electrical and plumbing systems, etc. Furthermore, even if the material is specified, the unit weights of elements reported in codes may vary from those given by manufacturers, and later use of the building may include some changes in dead loading. As a result, estimates of dead loadings can be in error by 15% to 20% or more.

Normally,thedeadloadisnotlargecomparedtothedesignloadforsimplestructuressuchas a beam or a single-story frame; however, for multi-story buildings it is important to have an accurate accounting of all the dead loads in order to properly design the columns, especially for the lower floors.

Liveloads

Live Loads can vary both in their magnitude and location. They may be caused by theweights of objects temporarily placed on a structure, moving vehicles, or natural forces. The minimum live loads specified in codes are determined from studying the history of their effects on existing structures. Usually, these loads include additional protection against excessive deflection or sudden overload.

BuildingLoads

The floors of buildings are assumed to be subjected to uniform live loads, which depend on the purpose for which the building is designed. These loadings are generally tabulated in local, state, or national codes.The values are determined from a history of loading various buildings. Theyinclude some protection against the possibilityof overload due to emergency situations, construction loads, and serviceability requirements due to vibration. In addition to uniform loads, some codes specify minimum concentrated live loads, caused by hand carts, automobiles, etc., which must also be applied anywhere to the floor system. For example, both uniform and concentrated live loads must be considered in the design of an automobile parking deck.

For some types of buildings having very large floor areas, many codes will allow a reduction in the uniform live load for a floor, since it is unlikelythat the prescribed live load will occur simultaneously throughout the entire structure at any one time.

HighwayBridgeloads

The primary live loads on bridge spans are those due to traffic, and the heaviest vehicle loading encountered is that caused by a series of trucks.

ImpactLoads

Moving vehicles may bounce or sidesway as they move over a bridge, and therefore they impart an impact to the deck.The percentage increase of the live loads due to impact is called the impact factor, I.This factor is generally obtained from formulas developed from experimental evidence.

Windloads

When structures block the flow of wind, the wind's kinetic energy is converted into potential energy of pressure, which causes a wind loading. The effect of wind on a structure depends upon the density and velocity of the air, the angle of incidence of the wind, the shape and stiffness ofthestructure, and theroughness ofits surface. Fordesign purposes, wind loadings can be treated using either a static or a dynamic approach.

Snowloads

In some parts of the country, roof loading due to snow can be quite severe, and therefore protection against possiblefailure is ofprimaryconcern. Design loadings typicallydepend on the building's general shape and roof geometry, wind exposure, location, its importance, and whether or not it is heated.

Earthquakeloads

Earthquakes produce loadings on a structure through its interaction with the ground and its response characteristics. These loadings result from the structure's distortion caused by the ground's motion and the lateral resistance of the structure. Their magnitude depends on the amount and type of ground accelerations and the mass and stiffness of the structure.

HydrostaticandSoilPressure

When structures are used to retain water, soil, or granular materials, the pressure developedby these loadings becomes an important criterion for their design. Examples of such types of structures include tanks, dams, ships, bulkheads, and retaining walls. Here the laws of hydrostatics and soil mechanics are applied to define the intensity of the loadings on the structure.

SupportConditions

Structural members are joined together in various ways depending on the intent of the designer. The three types of joints most often specified are the pin connection, the roller support, and the fixed joint. A pin-connected joint and a roller support allow some freedom for slight rotation, whereas a fixed joint allows no relative rotation between the connected members and is consequently more expensive to fabricate. Examples of these joints, fashionedinmetalandconcrete,areshowninFigs.5.3and5.4,respectively. Formosttimber structures, the members are assumed to be pin connected, since bolting or nailing them will not sufficiently restrain them from rotating with respect to each other.

Idealized models used in structural analysis that represent pinned and fixed supports and pin-connected and fixed-connected joints are shown in Figs. 5.5*a* and 5.5*b*. In reality, however,all connections exhibit some stiffness toward joint rotations, owing to friction and material behavior. In this case a more appropriate model for a support or joint might be that shown in Fig. 5.5*c*. If the torsional spring constant the joint is a pin, and if k : q, the joint is fixed.

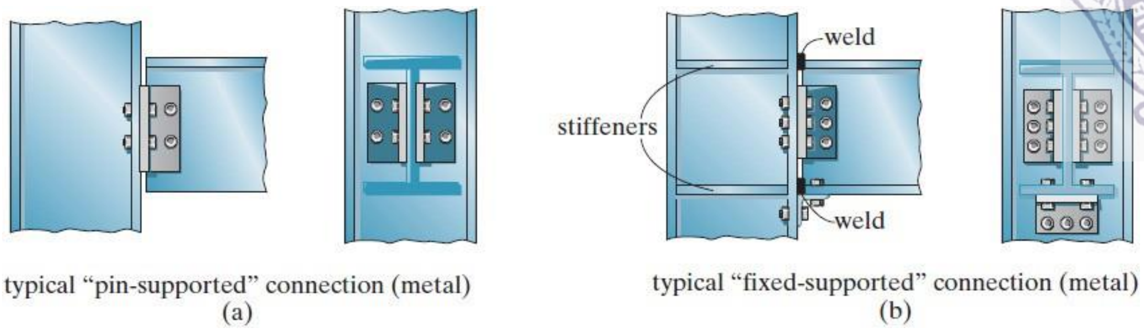


Figure5.3Typicalpinandfixedsupportedconnections

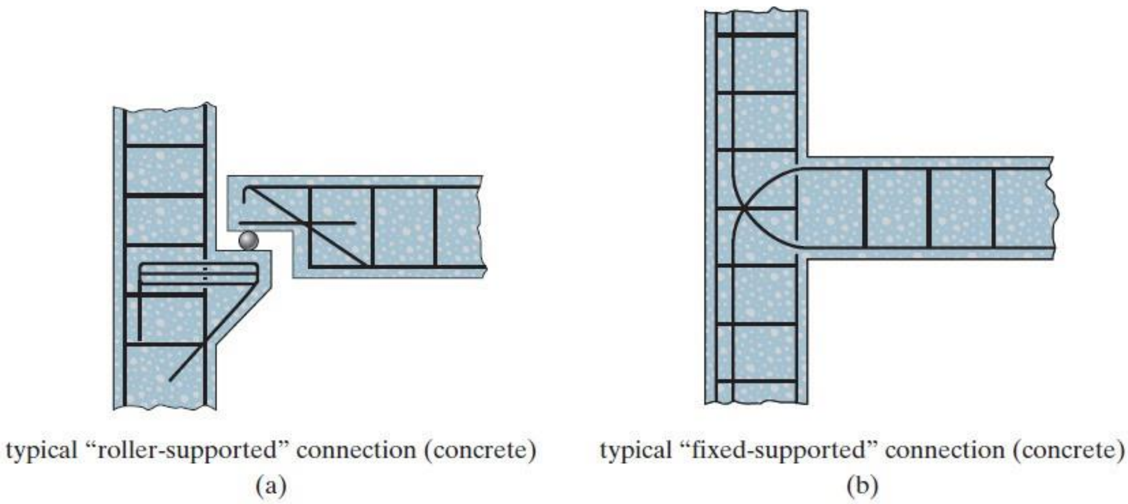


Figure5.4typicalrollerandfixedsupportedconnections

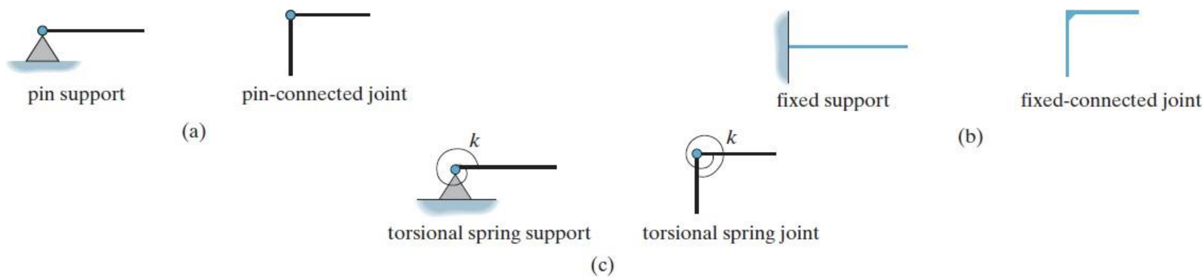


Figure5.5Varioussupportconditions

When selecting a particular model for each support or joint, the engineer must be aware of how the assumptions will affect the actual performance of the member and whether the assumptions are reasonable for the structural design. For example, consider the beam shown in Fig. 5.6a, which is used to support a concentrated load P. The angle connection at support A is likethat in Fig. 5.2a and can thereforebeidealized as atypical pin support. Furthermore, the support at B provides an approximate point of smooth contact and so it can be idealizedas a roller. The beam’s thickness can be neglected since it is small in comparison to the beam’slength,andthereforetheidealizedmodelofthebeamisasshowninFig.5.6b.The analysisoftheloadingsinthisbeam shouldgiveresultsthatcloselyapproximatetheloadings in the actual beam.

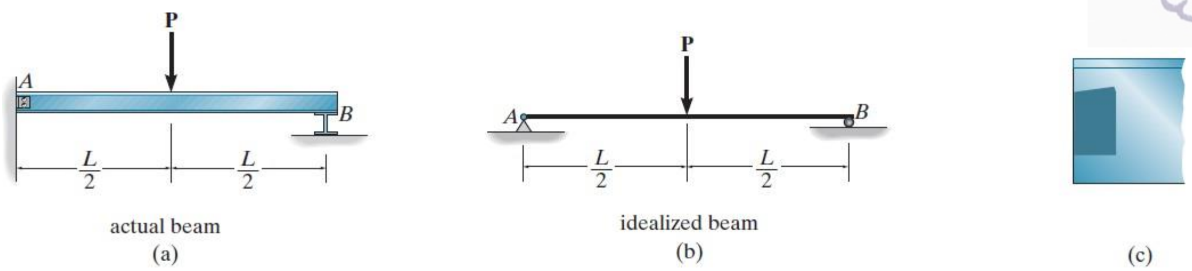


Figure5.6Actualandidealisedbeams

CalculationofReactions

Wefoundoutearlierthatifabodyorobjectofanysortisstationary,thentheforcesonit balance, as follows:

Total force upwards = Total force downwards
Totalforcetotheleft=Totalforcetotheright

Next we will find out how to use this information to calculate reactions – that is, the upward forces that occur at beam supports in response to the forces on the beam.

MomentEquilibrium

we found that if an object or body is stationary, it doesn't rotate and the total clockwise moment about any point on the object is equal to the total anticlockwise moment about the same point. This is the third rule of equilibrium.The three rules of equilibrium are expressed in Fig. 5.7

The three rules of equilibrium can be used to calculate reactions.A reaction is a force (usually upwards) that occurs at a support of a beam or similar structural element. A reaction counteracts the (usually downward) forces in the structure to maintain equilibrium. It is important to be able to calculate these reactions. If the support is a column, for example, the reaction represents the force in the column, which we would need to know in order to design the column.

Consider the example shown in Fig. 5.8. The thick horizontal line represents a beam of span6 metres which is simply supported at its two ends, A and B. The only load on the beam is a point load of 18 kN, which acts vertically downwards at a position 4 metres from point A.We are going to calculate the reactions *R*A and *R*B (that is, the support reactions at points A and B respectively).

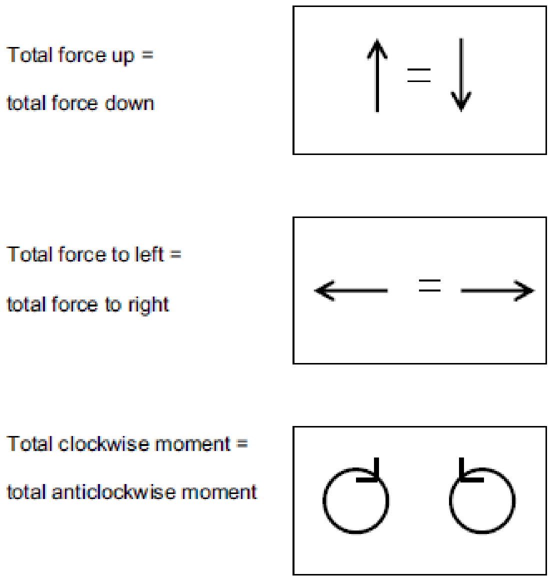


Figure5.7Therulesofequilibrium.